MONETARY MODEL

Version 2 - September 2020

Abstract

In order to build a global digital currency — that truly functions as such — trust and credibility are essential.

Like money, trust doesn't grow on trees. It needs to be earned. Credibility must be established.

Sögur's monetary model is designed to support the development of the currency from day one, when it is new and unproven, through its growth, until evolution into a standalone currency for widespread, everyday use.

This will be a long process, with multiple stages. In the life of a currency, some future events can be envisaged and even predicted. There are also unknowns, which we acknowledge from the outset.

This document is a blueprint of measures that we believe support the Sögur token, allowing it to gain trust. We outline core objectives for Sögur's economy, and declare our guiding principles, before establishing how to meet them.

We have rigorously tested our assumptions with Sögur's advisory board and team, and other prominent economists. Thanks to their input we've made several crucial improvements. We wish to further open our model to broader debate. Hoping to draw feedback and criticism, we now publish our updated monetary model.

To join the debate, we invite you to send comments and queries: economy@sogur.com

Note on Version 2 of this document

Since developing Sögur's monetary model over three years ago, the world, including the crypto ecosystem, has changed significantly. As a result, we are revising this model to stimulate the crypto community and incentivise early adoption while maintaining the fundamental features of SGR which support its evolution into a global currency.

To summarise the main changes to the revised monetary model:

1. A rapid reduction of the reserve ratio to 80% at a market cap of 12M SDR, after which the reserve ratio will continue to decrease slowly, until converging with the original model at a market cap of 1B SDR.

2. As a result of point 1, higher volatility in the initial market cap range generates unique opportunities for early adopters.

3. The price band starts at a width of 0.15% as in the original model, but increases from day one rather than rising only after reaching a market cap of 1B SDR. This change is possible since we now know there is already an efficient secondary market for SGR.

You can find more information about these changes in Appendix E.

1. INTRODUCTION

The Need for a Monetary Model — Currency Governance

The basis of today's prevailing currency system is first and foremost trust.

Objects of intrinsic value — food, clothes, etc. — are traded every day in return for objects with no intrinsic value, such as banknotes, coins or electronic money. These forms of money are based on the trust that this same representation of worth will be able to purchase things of similar value in the future. This is the basis of the fiat currency system.

In this system, the responsibility to maintain the stability of a currency's purchasing power is taken by central authorities. Without such governance, currencies would be left completely exposed to the vagaries of market sentiment.

Until now, the administration of currency has mostly been the purview of governments. However, there is no reason why governments should be the only entities to issue a currency.

In fact, currency could benefit from responsible competition. Alternative models can be presented in order to motivate progress; currency should have the chance to evolve to better suit the changing needs of global citizens.

Sögur offers an alternative model to state-managed fiat: a global currency, governed in monetary matters by code. The benefits of code are that it is transparent, predictable and impartial. Code does not favour any party, nor does it carry personal considerations. A global currency can be an appropriate means to serve our global society, in which economic activity is no longer restricted by national boundaries.

Sögur's new system first has to prove itself in the eyes of the public. Therefore, while Sögur is in the process of gaining acceptance, it draws trust from the existing monetary system. Sögur's economy begins with full backing by a reserve of major fiat currencies. Then, as the economy develops, our model reduces its anchoring to the existing system and starts to gain its own, independent value.

This paper contains a full account of Sögur's monetary model; governed by an autonomous smart contract, responsible for guiding the Sögur economy on a path of steady and appropriate growth, including incentives for adoption.. The Contract gradually and carefully shifts the source of trust from current dominant currencies to an eventual standalone value

Modelling Trust

Sögur tokens (SGR) are issued by the Sögur Smart Contract, which also offers to buy back and burn existing tokens.

Thus, the Contract adjusts the supply of Sögur tokens to meet market demand, and to limit the impact of fluctuations in market confidence on SGR price. When the economy expands, the Contract increases the supply of SGR, moderating price appreciation. When the economy shrinks, the Contract reduces the token supply, curbing price depreciation.

The proceeds of issuing new SGR tokens are kept in a reserve, held in reputable banks, and stored in liquid assets that replicate the currency composition of the SDR — a basket of fiat currencies (USD, EUR, etc.) created by the International Monetary Fund. A small portion of the Reserve is held in ETH buffer, that is used by the smart contract

to pay SGR sellers. In addition, for operational purposes some of the reserve might be held in licensed Electronic Money Institutions (“EMI”) and stablecoins, see chapter 11 below for more details.

The Reserve supports the Sögur economy; it provides the Smart Contract with the ability to buy back Sögur tokens when demand falls. The Reserve only provides a fractional backing; it does not have the full means to buy back all SGR tokens at current market price. The backing ratio decreases as the economy expands.

Thus, Sögur is a variable fractional reserve economy.

The fraction of Sögur’s economy backed by the Reserve represents the amount of trust drawn from the currencies that comprise the SDR currency basket. At the beginning of Sögur’s life, our model maintains a fully backed Reserve. One hundred percent of trust is obtained from the SDR currencies. Then, as the economy grows, the model begins to reduce its dependence on other currencies, and Sögur acquires its own amassed trust. The departure from full backing would gradually allow for market forces to affect the price of SGR, providing it with an increasing (although restrained) level of volatility and allowing SGR’s price to rise, thus incentivizing adoption of the currency.

2. SÖGUR CONTRACT AS SGR LIQUIDITY PROVIDER

A large part of this paper is devoted to describing Sögur's pricing model, which sets a range for SGR value based on the strength of the SGR economy.

The Sögur Contract ensures the value of SGR tokens is within the range given by the pricing model, by taking an active part in the SGR market, acting as a *liquidity provider*.

The Contract always offers to sell new SGR tokens, at a price — the ***ask price*** — given by Sögur's model. Conversely, the Contract offers to buy back and burn SGR tokens at the model's ***bid price***.

The bid and ask prices are not fixed — they increase when tokens are bought from the Contract, and decrease when tokens are sold back.

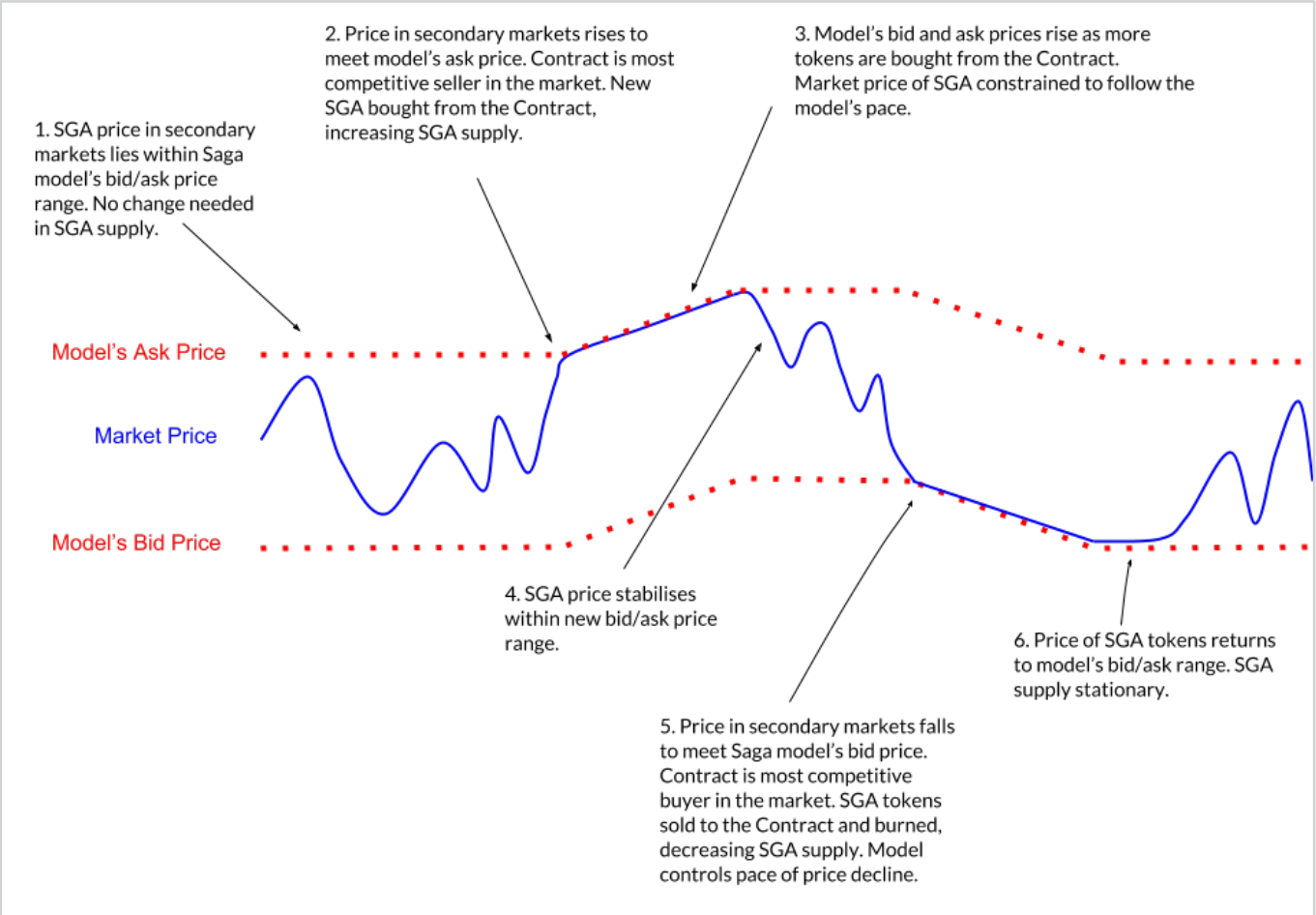
At any given moment the price of SGR tokens in the secondary market is constrained to lie within the pricing model's bid/ask range. Whenever the market price of SGR leaves this range, arbitrageurs will use the smart contract to mint or burn SGR tokens, resulting, in restrained price fluctuations.

Consider an example in which demand for SGR rises and the price of SGR in the secondary market becomes higher than the model's ask price. The Sögur Contract offers the cheapest price for buying SGR, so people (e.g. arbitrageurs) buy SGR directly from the Contract, not from the market. The supply of SGR rises in accordance with increased demand.

SGR will continue to be bought from the Contract for as long as the Contract offers the most competitive price; until the model's ask price has risen to meet the secondary market price.

Thus, at all times the Sögur model either determines a range for SGR price, or controls the pace of movement when SGR price moves beyond this range. Sögur's model imposes different rates of price movement at different stages of the economy's development.

Figure 1: Sögur Contract as SGR liquidity provider



For the sake of simplicity, the majority of this paper considers a pure pricing model, where the bid price is equal to the ask price (i.e. a bid/ask spread of zero). We can therefore refer to the 'price' of Sögur tokens without ambiguity and assume the model completely dictates the price of SGR.

We designed Sögur's pricing model using the same framework; it is more convenient to design the model to fulfil our aims if we assume it completely sets SGR price.

In Chapter 10 we describe the full framework, which includes a bid/ask price spread.

3. OBJECTIVES FOR SÖGUR'S MONETARY MODEL

Sögur's monetary model has several objectives:

1. To prevent the possibility of malicious users manipulating the system and gaming the Reserve for unfair profit.

2. To ensure the pricing model is 'robust': if all SGR holders decide to sell their tokens back to the Contract, there must be enough money in the Reserve to pay them back at the model's prices.

3. The price of SGR should reflect the strength of the Sögur currency. This could be measured by metrics such as: i) the amount of SGR in circulation; ii) the number of unique SGR holders; iii) the volume of SGR transactions.

4. The monetary model should aim to regulate volatility of SGR price, while also balancing the need for sufficient price appreciation.

5. Support the growth of Sögur's economy by incentivising early adopters of the currency.

Objectives 1 & 3 seem somewhat discordant. For example, imagine a pricing model that takes into account the daily volume of transactions between SGR participants. Two malicious participants could join forces and do the following: both buy SGR from the Contract, then trade large amounts of SGR between themselves, causing inflation in SGR price. Selling their SGR back to the Contract, they will realise an unfair profit.

An effective way of ensuring the model is not susceptible to unfair gaming by participants is to make it path-independent. The behaviour of the model in a particular state should depend on that state alone, not how we got there. It is not possible to add more than one free variable to a pricing function and at the same time have a model which is both path-independent and robust.

Therefore, instead of designing a model that considers several metrics of Sögur's health, we built a model that focuses on just one: the size of the economy.

Sögur's pricing function is an increasing function of the SGR economy. Natural ways of measuring the size of the economy are: the total number of SGR in circulation; the amount of money in the Reserve; the market cap of SGR.

Sögur's pricing model works as follows:
The price that the Contract charges to issue a new SGR token depends only on the number of existing tokens in circulation (i.e. discounting tokens sold back to the Contract and burned). The first SGR token is sold[1] at price $P(1)$. The second is sold at price $P(2)$, where $P(2) \geq P(1)$.

In general, the N^{th} token is sold for $P(N)$, where $P(N)$ is at least as large as the price of the previous token sold.

[1]We have simplified things here slightly for ease of understanding. In practice we consider SGR to be bought in infinitesimal pieces, thus the correct price to pay for a SGR token involves taking the integral of the pricing function. Exact details given in Chapter 5.

Proceeds from the sale of SGR tokens are kept in the Sögur Reserve. When SGR is sold back to the Contract, the seller is reimbursed at the current price, regardless of when the token was originally purchased. So if the token is sold when there are N tokens in circulation, the seller receives $P(N)$ from the Contract in return.

Taking this approach, objectives 1 and 2 are automatically fulfilled:

1. Path independence means the Reserve cannot be gamed. There is nothing to be gained from buying and immediately selling SGR tokens.

2. The Reserve remains solvent at all times. There are always enough funds to reimburse SGR tokens in the market. The Reserve simply withdraws money deposited into it at the point of the last token purchase.

Objective 3 — the accurate reflection of Sögur's strength by SGR price — is fulfilled, as much as possible, by having Sögur's pricing function determined by the size of Sögur's economy.

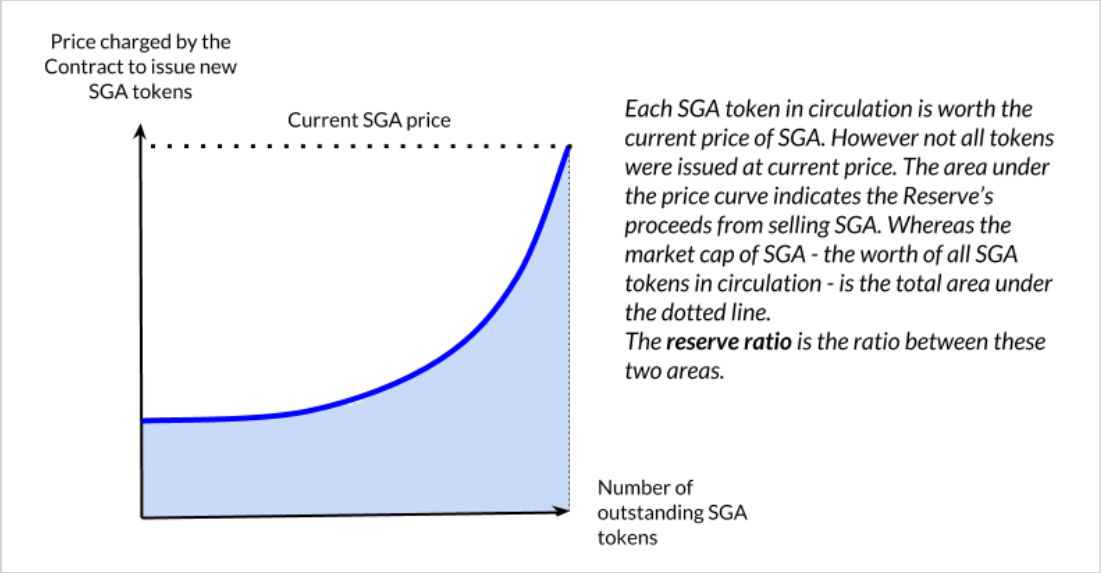
The remainder of this paper addresses the 4th, most intricate aspect of Sögur's monetary model: taming volatility while supporting growth as Sögur's economy develops. This will also meet the 5th objective of providing sufficient incentives for adoption and supporting the creation of sustainable growth..

4. THE RESERVE RATIO

Sögur's pricing model increases the price of SGR as more tokens are bought from the Contract. However, SGR tokens are always all worth the current price of SGR, regardless of when the tokens were issued, or how much was paid for them. The result is that the, once the reserve ratio decreases below 100%, Sögur Reserve contains less money than the market cap of SGR

We define the **reserve ratio** to be: *the amount of money in the Reserve as a percentage of Sögur's market cap*. For example, a reserve ratio of 20% means the market cap is five times the value of the Reserve.

Figure 2: Visual description of the reserve ratio



The reserve ratio has several intuitive interpretations:

1. It is a measure of the ability of the Contract to buy back SGR tokens, without having to reduce the price of SGR.

Consider a case in which the reserve ratio is 20%. The Reserve only has 20% of the funds needed to buy back all SGR tokens at current price. If everyone begins selling their tokens back to the Contract, SGR price must drop for the Contract to pay everyone back.

If the reserve ratio is higher, say 80%, the Contract must still reduce the price of SGR as participants exit the economy, but the reduction in price need not be so drastic.

2. The reserve ratio represents the average price that buyers have so far paid for SGR, in relation to the current price. For example, a reserve ratio of 20% means the average buyer bought SGR from the Contract for 80% less than the current value.

3. Equally, the reserve ratio represents the average price that sellers of SGR receive, should they all sell SGR back to the Contract.

4. The reserve ratio is a measure of the rate of SGR price change. If SGR price grows rapidly with the number of SGR tokens, the reserve ratio will be low.

5. Perhaps most importantly, (one minus) the reserve ratio represents the trust and inherent value embedded in Sögur.

Reserve Ratio & Market Trust

The value of an SGR token is derived from two distinct sources:

1. The Reserve backing.

2. 'Market trust' — determined by less tangible ingredients, such as market confidence and the general perception of SGR's utility as a currency.

While the value derived from the Reserve is stable[2], market trust is likely to be volatile, especially when Sögur's economy is small. On the other hand, market trust can grow against the SDR benchmark, whereas the Reserve — by definition — cannot.

The Reserve, held in major currencies that make up the SDR basket, represents Sögur's anchoring to the established financial system. The reserve ratio therefore describes the *extent* to which Sögur's economy draws value from existing currencies, or rather has its own independent value, reflecting market trust.

Reserve Ratio & Volatility

One of our key objectives is to mitigate SGR price volatility. However, we also believe SGR price should vary to accurately reflect the strength of Sögur's economy.

Allowing the price of SGR to change opens the door to volatility. As market demand for SGR fluctuates, participants buy or sell SGR against the Contract, and the price of SGR — given by the pricing model — changes.

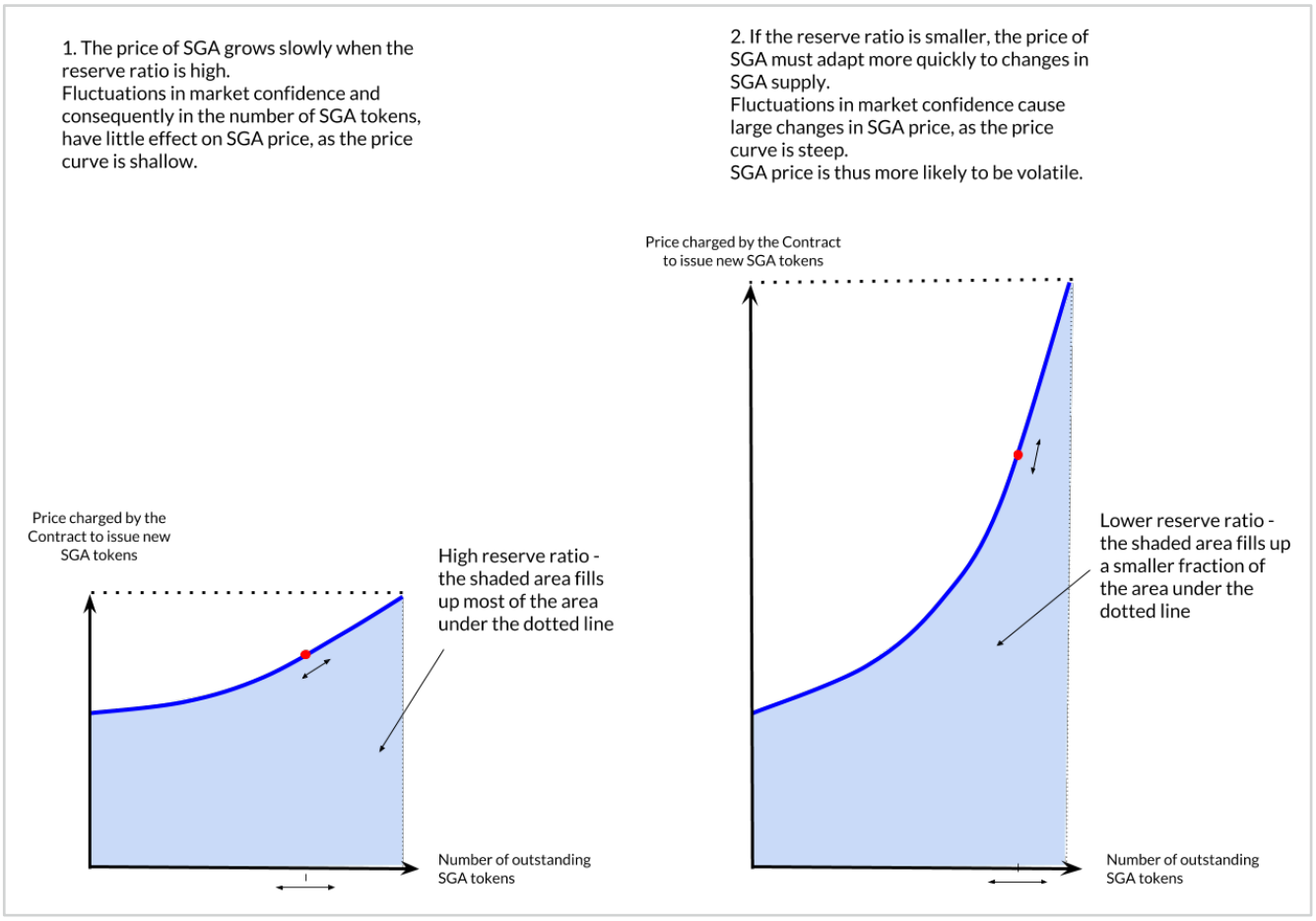
The steepness of Sögur's pricing curve dictates the exposure of SGR price to volatility. If the curve is shallow, the Contract provides a large amount of resistance to volatility — i.e. a large number of SGR must be traded before a significant effect on SGR price is seen. Conversely, if the price curve is steep, SGR price is more susceptible to volatility.

As mentioned above, the reserve ratio is precisely what determines the steepness of the pricing curve.

Figure 3: Reserve ratio & Volatility
Market forces determine the number of outstanding SGR tokens. The model's pricing curve

[2] In terms of the unit of account, the SDR, at a given time.

determines how changes in the number of SGR tokens affect SGR price.



From Reserve Ratio Model to Pricing Model

When designing Sögur's pricing curve we could, in theory, pick any arbitrary increasing function. However, it is difficult to choose a pricing function with any real meaning in an economic sense.

A better approach is to choose what the **reserve ratio** should be at various stages of Sögur's growth. Indeed, the reserve ratio has an intuitive interpretation; it is a measure, amongst other things, of how much independent trust Sögur's economy has. Furthermore, it is a measure of the power of Sögur's Reserve to dampen SGR price movement and thus volatility.

Once we have designed how the reserve ratio should vary, a pricing function that creates this exact behaviour in the reserve ratio can be derived mathematically, as shown in the next chapter.

The pricing function is then not arbitrary; it is a derivative of the reserve ratio model, based upon economic considerations.

Sögur's Reserve Ratio Model

Sögur's monetary model is based on a variable reserve ratio. The reserve ratio starts at 100% and then gradually decreases as Sögur's economy grows.

Reasons for this are as follows:

When Sögur's economy is small, market trust is low, and therefore unstable. In order to limit SGR price exposure to fluctuations in market trust, a high reserve ratio is employed, favouring stability over growth at this stage. However, we do not wish to eliminate volatility altogether, as a certain level of price fluctuation is a key feature of any currency (including national currencies, which constantly fluctuate against each other). With a new currency, which needs to incentivise and promote adoption, this is even more crucial for generating growth

As Sögur's economy becomes stronger, market trust (and consequently SGR's independent value) increases and becomes more stable. At this point, a smaller portion of SGR's value should be based on the Reserve. In other words, a lower reserve ratio can be used, allowing greater exposure to market forces in favour of faster growth.

Sögur's monetary model applies a reserve ratio of 100% at the very beginning of Sögur's economy. The reserve ratio declines starting at a market cap of 2M SDR. As a result, SGR price is connected to its economic success and adoption. From this point, the reserve ratio is slowly lowered until it eventually reaches a minimum of 10%, then remains constant. A 10% reserve ratio is identical to the US Federal Reserve's current required reserve ratio on banks' Net Transaction Accounts.

5. THE MATHS BEHIND SÖGUR'S PRICING MODEL

We start by defining some notation:

R — the value of the Reserve

r — the reserve ratio

N — the number of outstanding SGR tokens

P — the price of SGR as given by Sögur's pricing model

Deriving Price from Reserve Ratio

Recall that we designed Sögur's pricing model by deciding how the reserve ratio should vary as the economy develops. Once we selected how the reserve ratio should behave, a pricing function that creates this behaviour can be derived as follows:

The reserve ratio, by definition, is the ratio between the amount of money in the Reserve and the market cap of Sögur.

(1) $$R = rNP$$

Thus $P = \frac{R}{rN}$. This is not yet enough, as we need to know R as a function of N.
Fortunately, our variables satisfy another equation:

(2) $$R(N) = \int_0^N P$$

This equation says the amount of money in the Reserve — when there are N tokens in circulation — is precisely the proceeds from selling those tokens.

Combining the two equations, we get: $rNP = \int P$

Differentiating and rearranging gives:

(3) $$\frac{1}{P} \cdot \frac{dP}{dN} = \frac{1}{rN}(1 - r - N\frac{dr}{dN})$$

We then solve this differential equation to get P as a function of N.

We consider SGR to be bought in infinitesimal pieces, so the cost of buying m new tokens when there are already N tokens in circulation is given by $\int_N^{N+m} P(n)\, dn.$

Sögur's Reserve Ratio Function

The reserve ratio in Sögur's model decreases as a piecewise linear function of N — the number of SGR tokens in circulation. Under this choice differential equation (3) is easily solved. Furthermore, we do not lose anything by limiting ourselves to using a piecewise linear function for r, since piecewise linear functions can approximate any continuous function, to any degree of accuracy[3].

In a given interval, i, the reserve ratio can be expressed as: $r(N) = \alpha_i - \beta_i N$ where α_i, β_i are non-negative constants. The solution to differential equation (3) is then:

$P = \frac{\omega_i}{N \cdot r} \cdot \left(\frac{N}{r}\right)^{1/\alpha_i}$ where ω_i is a constant that ensures price continuity across intervals.

And:

$R = \omega_i \cdot \left(\frac{N}{r}\right)^{1/\alpha_i}$ (using equation 1)

The starting price of SGR (the initial condition of the differential equation) is free for us to choose — we chose it to be 1 SDR.

The full details of how we designed the shape of the reserve ratio function can be found in Appendix A.

Reserve Ratio & Volatility — Worked Examples

We give mathematical backing to our claim that the reserve ratio specifies the extent to which the Contract can restrain SGR price volatility. We do this by considering the cash flow needed to move SGR price.

Recall that the price of SGR changes when capital is injected into — or removed from — the Sögur Reserve. If a substantial cash flow is needed to change SGR price significantly, then the Contract provides a high level of protection against market volatility. If a small net change in the Reserve results in a large price change, then SGR price will be more sensitive to volatility.

For simplicity in our calculations below, we consider a constant reserve ratio, though similar principles apply when working with a variable reserve ratio.

From equation (3) above we can derive that if the reserve ratio is kept at a constant value, r, then the relation between the price of SGR and the value of the Reserve is given by: $P \sim R^{1-r}$.

Differentiating gives that $\frac{dP}{dR} = (1-r) \cdot \frac{P}{R}$, which can also be written as $\frac{dP}{P} = (1-r) \cdot \frac{dR}{R}$.

[3]In mathematical terminology, the fact that the set of piecewise linear functions is dense in the space of continuous functions on a closed, bounded interval under the uniform metric.

This states that a percentage change in the Reserve value causes a percentage change in SGR price reduced by a factor of $1 - r$. For example, if the model maintains a low reserve ratio of $r = 20\%$, then a 10% change in the Reserve would cause a price change in the order of 8%. If a higher reserve ratio is used, say $r = 80\%$, then the same10% change in the Reserve only causes a 2% change in SGR price.

Further analysis can be found in Appendix B, where we use Monte Carlo simulations to gauge the sensitivity of SGR price to volatility, at various points of Sögur's economy development.

6. SAGA GENESIS

The resources to build Sögur's ecosystem derive from proceeds raised from early contributors, who support Sögur before the SGR economy launches. They should receive commensurate compensation for that risk. On the other hand, compensation should not affect Sögur's economy adversely.

Sögur cannot just give out the first SGR tokens for free to early backers: the Sögur Contract is committed to act as a liquidity provider for SGR tokens. It must ensure that there is always enough money in the Reserve to buy back SGR tokens at the price set by its pricing function. Therefore, the SGR Contract cannot mint SGR tokens before there is money in the Reserve. The same goes for selling SGR tokens at a discount — as other digital currencies have done.

Therefore, we devised a different way to recompense early backers, using a product called **Saga Genesis**.

When designing Saga Genesis, we had the following objectives:

1. The value of Saga Genesis should be tied to the success of Sögur.
2. Saga Genesis should offer high prospect in return for the high risk taken by early backers.
3. The Saga Genesis model should not have an adverse effect on the main Sögur economy model.
4. Saga Genesis holders should not hold too much power.
5. The lifetime of Saga Genesis should be limited — eventually the Saga Genesis economy should merge with the Sögur economy.

We came up with a framework that recompenses early investors with SGR tokens, but not immediately. Instead, they receive their SGR tokens gradually, only if and when Sögur's economy grows through various milestones.

The exact mechanism for Saga Genesis is as follows:

A Saga Genesis (SGN) is a digital token that represents a *potential* to receive up to 7 SGR tokens. The Sögur model defines several market-cap milestones, which we call **'Genesis Minting Points'.** When the Sögur economy reaches a Minting Point for the first time, new SGR tokens are generated on behalf of each SGN token. Overall, across all the Minting Points, 7 SGR tokens are minted for each SGN token.

A Saga Genesis token holder can at any time send their SGN token to the Saga Genesis Contract and receive the SGR tokens that have been minted for the SGN token. We call this 'converting SGN to SGR'. However, in doing so, the SGN token is burned. So if the SGN token was converted before all 7 SGR tokens were minted, the holder loses the potential to receive the remaining SGR tokens.

This framework for Saga Genesis fulfilled our objectives:

1. **The value of SGN is tied to the success of SGR:** The SGR-SGN **conversion ratio** — the number of SGR received when one SGN token is converted — depends on the number of market-cap milestones the SGR economy has grown through i.e. SGN investors' return only increases if the Sögur economy itself flourishes. The *value* of the SGR tokens received depends on the price of SGR, which is also dependent on the strength of the Sögur economy.

2. **SGN represents a high-risk, high-prospect investment:** If the Sögur economy fails to pass many (or any) of the market cap milestones, SGN will have a low (or no) return. On the other hand, if the economy succeeds, an SGN token can be worth up to 7 SGRs, yielding a significant return.

3. **Saga Genesis does not adversely affect the main SGR economy:** Generating SGR tokens for SGN holders, without adding extra money into the Reserve, must impact on Sögur's pricing model. The size of the impact depends on the ratio between the number of SGR tokens generated and the total number of SGR tokens.

   Since we mint SGR tokens for SGN holders at *market cap* milestones rather than at *time* milestones, we can ensure that we only generate SGR tokens for SGN holders when the Sögur economy can handle it. We designed the location of each Genesis milestone, and the number of SGR generated, to contain the effect on the SGR economy.[4]

   An additional benefit of minting at market cap milestones is that the model is deterministic; if we generated SGR tokens at fixed points in *time*, we would have no way of knowing in advance the effect this would have on the Sögur economy.

4. **Saga Genesis holders do not have too much power:** There are a limited number of Saga Genesis tokens, 107 million overall. Consequently, the number of SGR tokens minted for early investors is limited.

   Moreover, these SGR tokens are minted in stages — only if and when the Sögur economy has grown through a market cap milestone. This means the number of SGR tokens owned by early investors can only increase once the number of SGR tokens bought by regular participants has also increased. So early investors never swamp the main SGR economy.

   This is a significant benefit of Sögur's model. Usually, when a new company or digital currency is created, early backers start by owning 100% of all the equity or tokens. This percentage decreases when equity is sold or more tokens are generated. However the decrease is often gradual, so early backers retain significant control.

   Take Ethereum. Approximately 70 million ETH were generated for early backers and participants of the pre-sale. Since then, to date, (August 2020), roughly 40 million new ETH have been generated, meaning that the original tokens still account for over 60% of the ETH economy.

   By contrast, in Sögur's model, early backers start out owning no SGR tokens; and at their maximum, the total amount of SGR token created as part of the SGN mechanism accounts for less than 30% of then outstanding all SGR tokens.

5. **The influence of Saga Genesis on Sögur's main economy is limited:** Once all the Minting Points have been reached, and 7 SGR tokens have been minted for each SGN token, Saga Genesis ceases to impact Sögur's pricing model. Saga Genesis becomes synonymous with Sögur — an SGN token is just like a 15 SGR bill.

[4] Exact details of how we designed Genesis Minting Points can be found in Appendix A of this paper.

Remarks

- Those who convert SGN to SGR before all 7 SGR tokens have been minted lose their rights to the remaining SGR tokens. This protects the Sögur economy; it encourages SGN holders to stay in the Sögur economy, not just take their SGR tokens and 'run'.

- We assume a secondary market for SGN tokens may evolve, which will allow SGN holders to sell them at a price reflecting the possibility that more SGR tokens will be minted for the SGN token, whereas converting SGN with our contract negates this possibility. Sögur has no interest in such secondary market, which will have no effect over the economy and will be rendered irrelevant once all 15 SGR tokens have been minted.

- Secondly, when SGN is converted to SGR the rights to SGR tokens not yet minted are lost — but the SGR tokens are still generated at Minting Points. This ensures Sögur's pricing model is deterministic. The alternative would be that each time the Sögur economy reaches a Genesis Minting Point, the number of SGR tokens minted — and consequently the effect on the pricing model — would depend on the number of remaining SGN tokens. We rejected this option; we do not want the actions of SGN token holders to impact Sögur's pricing model, since this could open the door to manipulation.

- One final point to note is that the conversion ratio of SGN-SGR depends on the number of Genesis Minting Points the Sögur economy has passed. In other words, the conversion ratio depends on the *high-water mark* of Sögur's economy.

  This raises one potential problem: the possibility that SGN holders may attempt to artificially increase Sögur's high-water mark by buying and immediately selling SGR tokens against the Sögur Contract. To protect against this, the Contract only mints new SGR tokens for SGN holders if Sögur's market cap remains above a Genesis Minting Point for a period of seven days.

Saga Genesis & the Reserve Ratio

When Sögur's economy reaches a market cap at which a Genesis Minting Point occurs, the Contract generates SGR tokens for SGN holders. This is done without any increase to the Reserve.

Recall that the reserve ratio is defined as $r = \frac{R}{NP}$

At a Minting Point, N increases with R remaining the same. The price, P, of SGR should also remain the same, so SGR holders are not immediately disadvantaged by the minting of new tokens. Therefore the reserve ratio drops. If there are N SGR tokens in circulation before the Genesis Minting Point, and n new tokens are minted, the new reserve ratio will be: $r_{post-minting} = \frac{N}{N + n} \cdot r_{pre-minting}$

Once SGR tokens have been minted for SGN holders (after a week has passed), it cannot be undone: if the Sögur economy begins to shrink, we do not un-mint the tokens. Therefore we need a new model for when the economy shrinks backwards through a Minting Point milestone.
We defer the discussion of a 'Shrinking Economy' to a later chapter of this document.

Sogur Monetary Technologies

Sogur Monetary Technologies is an English limited by guarantee company, working under non-for-profit principles and dedicated to creating and maintaining the Sögur economy. The company owns 36% of SGN tokens. These will be used to fund the company's activities once proceeds from the original SGN sale have run out. As the company is the most dominant SGN holder, a vesting mechanism has been included in Sögur's model to prevent the company from selling its SGN tokens during early stages of Sögur's development.

SGN tokens belonging to Sogur Monetary Technologies are not all created at the beginning of Sögur's economy. Some are created only if the Sögur economy reaches various points, which we call Vesting Points. There are three Vesting Points in Sögur's model, and at each, a portion of the company's SGN tokens are created and SGR tokens are also minted on their behalf to catch them up with the other SGN tokens. Delaying the minting of the company's SGR tokens to a later stage minimises the impact on the economy.

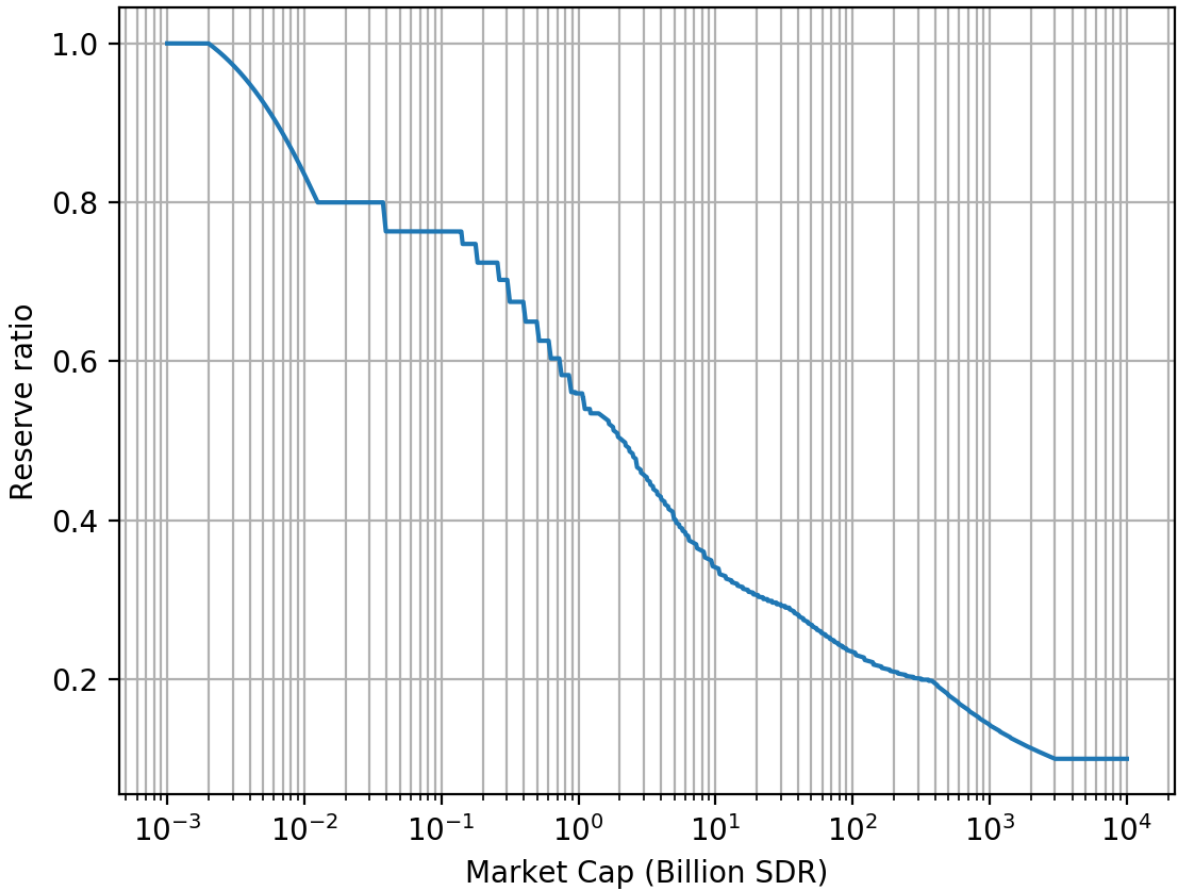
7. SÖGUR'S MODEL IN FIGURES AND TABLES

We built Sögur's monetary model based on the principles detailed above. Here we show the main features of the model. Appendix A lists the exact parameters used to build the model.

Reserve Ratio Function

The reserve ratio decreases as Sögur's economy increases in market cap. The decrease is linear in the number of SGR tokens in circulation. Drops to the reserve ratio also occur each time the economy reaches a Genesis Minting Point.

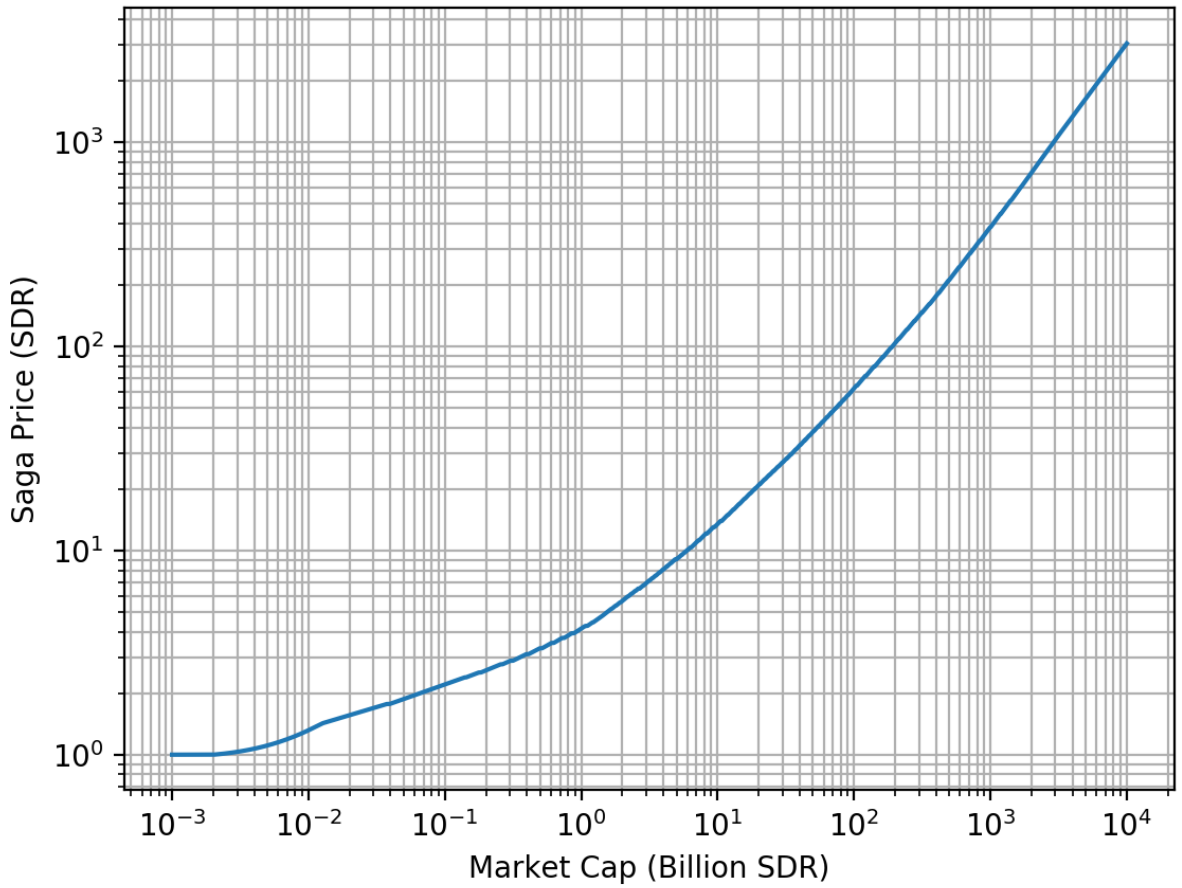
Figure 4: Reserve ratio as a function of Sögur's market cap
Market cap is shown on a logarithmic scale



Pricing Function

We now show how the price of SGR increases as Sögur's economy grows. The pricing function was calculated to create the exact behaviour in the reserve ratio shown in Figure 4.

Figure 5: SGR price as a function of SGR market cap



The following table contains an overview of Sögur's model.

Table 1: Key features of Sögur's economy at various market cap milestone.

Market Cap	SGR Price	Reserve Ratio	Total Funds in Reserve
(SDR)	(SDR)	%	(SDR)
1M	1.00	100.0%	1M
2M	1.00	100.0%	2M
3M	1.03	97.3%	2.9M
5M	1.11	92.6%	4.6M
7M	1.19	88.6%	6.2M
10M	1.32	83.5%	8.3M
20M	1.56	80.0%	16M
30M	1.69	80.0%	24M
50M	1.87	76.4%	38.2M
70M	2.03	76.4%	53.4M
100M	2.21	76.4%	76.3M
200M	2.59	72.4%	144.8M
300M	2.88	70.3%	210.7M
500M	3.32	65.0%	322.6M
700M	3.68	60.4%	422.6M
1B	4.15	56.0%	560M
2B	5.66	50.3%	1.006B
3B	6.93	45.7%	1.37B
5B	9.07	40.1%	2.007B
7B	10.99	37.1%	2.598B
10B	13.50	34.1%	3.4B
20B	20.92	30.6%	6.1B
30B	27.04	29.4%	8.8B
50B	38.19	26.9%	13.43B
70B	48.15	25.0%	15.311B
100B	62.30	23.5%	23.4B
200B	103.71	21.0%	41.9B
300B	141.36	20.1%	60.3B
500B	211.88	18.0%	90.0B

Reserve Ratio, Volatility & Growth

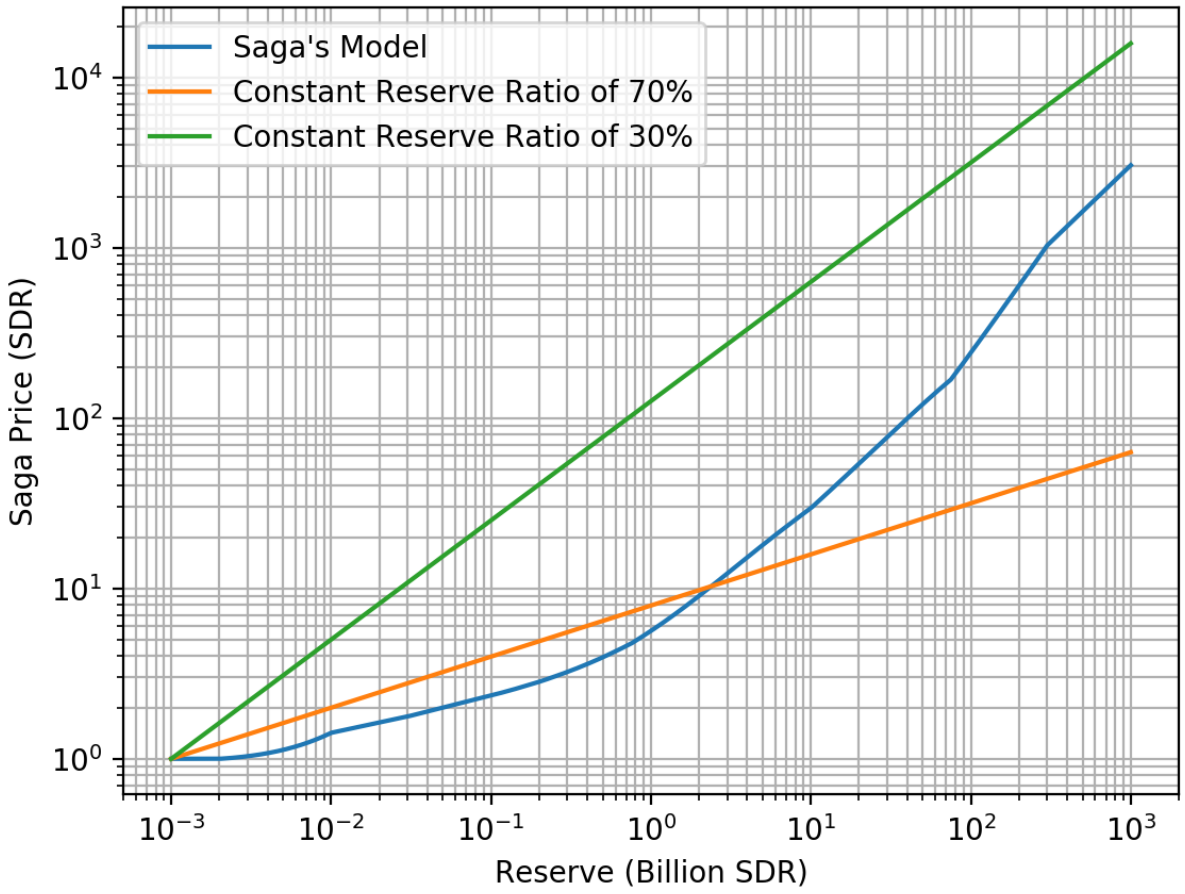
The Sögur Contract moderates the growth of SGR price. We claimed above that the reserve ratio gives a heuristic of the extent to which the Contract provides resistance to price change. When the model maintains a high reserve ratio, volatility and growth are discouraged; when the model maintains a low reserve ratio, SGR price is more open to volatility and growth.

Sögur's model keeps a relatively high reserve ratio during the early stages of the economy, in order to aid price stability. As Sögur's economy progresses, a lower reserve ratio is maintained, to allow faster growth.

The following figure compares SGR price growth of Sögur's model, to price growth of alternative models that employ constant reserve ratios.

Figure 6: Price growth of Sögur's model & two models that employ a constant reserve ratio

The model that keeps a constant 'low' reserve ratio exhibits the fastest price growth. The model with a 'high' reserve ratio exhibits slower growth. In Sögur's model, price growth is variable: at the beginning, growth is slow in order to encourage stability; as Sögur's economy grows our model reduces its resistance to price movement.



A good way of measuring sensitivity of SGR price to market volatility is to measure how much money must be injected into — or removed from — the Reserve in order for SGR price to move. Table 2 gives the sensitivity of SGR price at a selection of different market caps.

Note that as Sögur's economy grows and the model employs a lower reserve ratio, SGR price becomes more sensitive to change in *relative* terms — a smaller *percentage* change in the Reserve value gives rise to the same percentage price change.

However, since the Reserve becomes larger as the economy grows, it actually becomes harder to move SGR price in terms of the *absolute* amount of capital needed.

In other words, when the economy is large, the sheer size of the Reserve protects SGR price from volatility. We can allow the model to employ a smaller reserve ratio in order to give market forces a greater role in determining SGR price.

Table 2: Changes in Reserve needed to change SGR price by 1% & 10%

Market Cap	Reserve Value	SGR price	Reserve ratio	Relative Change in Reserve needed to increase SGR price by:		Amount of capital needed to increase SGR price by:	
				1%	10%	1%	10%
(SDR)	(SDR)	(SDR)	(%)	(%)	(%)	(SDR)	(SDR)
5M	4.63M	1.11	93%	4.80%	44.96%	222K	2.1M
10M	8.35M	1.32	84%	2.57%	31.81%	214K	2.65M
50M	38.2M	1.87	76%	4.30%	49.63%	1.65M	19M
100 M	76.3M	2.21	76%	4.30%	48.84%	3.3M	37M
500 M	322M	3.32	65%	2.70%	28.35%	8.7M	91M
1 B	560M	4.15	56%	2.29%	23.30%	12.8M	130M
5 B	2B	9.07	40%	1.37%	13.79%	27.45M	277M
10 B	3.4B	13.50	34%	1.32%	13.44%	45M	458M
50 B	13.4B	38.19	27%	1.15%	11.56%	155M	1.55B
100 B	23.4B	62.30	23%	1.13%	11.39%	265M	2.67 B
500 B	90B	211.88	18%	0.79%	7.78%	710M	7.02 B
1 T	142B	383.30	14%	0.76%	7.53%	1.09B	10.7 B

8. SHRINKING ECONOMY

Every time Sögur's economy reaches a new 'Genesis Minting Point' (hereafter GMP), the Contract mints SGR tokens for SGN holders. The process is not reversible — if Sögur's economy subsequently shrinks back through the GMP, the SGR tokens are not burned. Consequently, a new pricing model must be used — the economy cannot shrink in the exact same manner it grew.

The new model must satisfy the basic mathematical constraints that we presented in Chapter 5. An additional constraint also applies here: the new model must be consistent with the old. The amount of money in the Reserve when the economy reaches the GMP should be precisely the correct amount required to buy back all SGR tokens under the new model.

Mathematically, this constraint can be expressed as follows. If the previous pricing function was $P(N)$, and our new pricing function is $P'(N)$, then the following relation must hold:

$$\int_0^N P(m)\,dm = \int_0^{N+n} P'(m)\,dm$$

where N is the number of SGR tokens in circulation at the GMP, and n is the number of SGR tokens minted.

As long as we respect these constraints, in theory, we may design the new pricing function in any manner we choose.

Shrinking Economy Framework

Recall we designed the 'base pricing model' — the model that dictates how Sögur's economy grows — by focusing on the behaviour of the reserve ratio function. Recall also that the reserve ratio function fully determines the entire pricing model.

When designing models for a shrinking Sögur economy, we again focused on determining the reserve ratio function. We did so with reference to the original, 'base' function.

Our approach: the reserve ratio function for a shrinking economy should be higher than in the base model. Indeed, the reserve ratio is a measure of how much market trust Sögur's economy has earned; an economy going through a recession enjoys less trust than a growing economy of the exact same size. In addition, a higher reserve backing provides extra support to Sögur's shrinking economy — this should alleviate panic and help stimulate a recovery.

The approach we took is as follows:

Consider a GMP.

- Let r_{min} denote the value of the reserve ratio immediately after the GMP.
- Let $r_{base}(R)$ be the value of the reserve ratio in the base model when the Reserve value was R SDR.
- Let $r_{shrinking}(R)$ be the value of the reserve ratio in the new model, at this same point.

Sögur's model employs the following formula for $r_{shrinking}(R)$:

$$r_{shrinking}(R) \;=\; r_{base}(R) \;+\; (1 - r_{base}(R)) \cdot \frac{r_{base}(R) \;-\; r_{min}}{1 \;-\; r_{min}}$$

Notes:

1. The rightmost term is always positive, so $r_{shrinking}(R) \;\geq\; r_{base}(R)$ – i.e. the reserve ratio when the economy shrinks is always at least as large as when the economy grew.

2. The term $(1 - r_{base}(R)) \cdot \frac{r_{base}(R) \;-\; r_{min}}{1 \;-\; r_{min}}$ can be thought of as the amount of trust lost in Sögur's economy. It is proportional to how far the economy has shrunk since its high-water mark.

Figures 7-10 below give an illustration of what happens at a Minting Point, and what happens if the economy begins to shrink afterwards.

Figure 7: Illustration of SGR price around a GMP

The economy grows along the blue line until it reaches a GMP. At this point, new SGR tokens are minted. If the economy now begins to shrink it cannot shrink backwards along the blue line — the SGR tokens minted at the GMP cannot be un-minted. Instead, the economy shrinks along the green line.

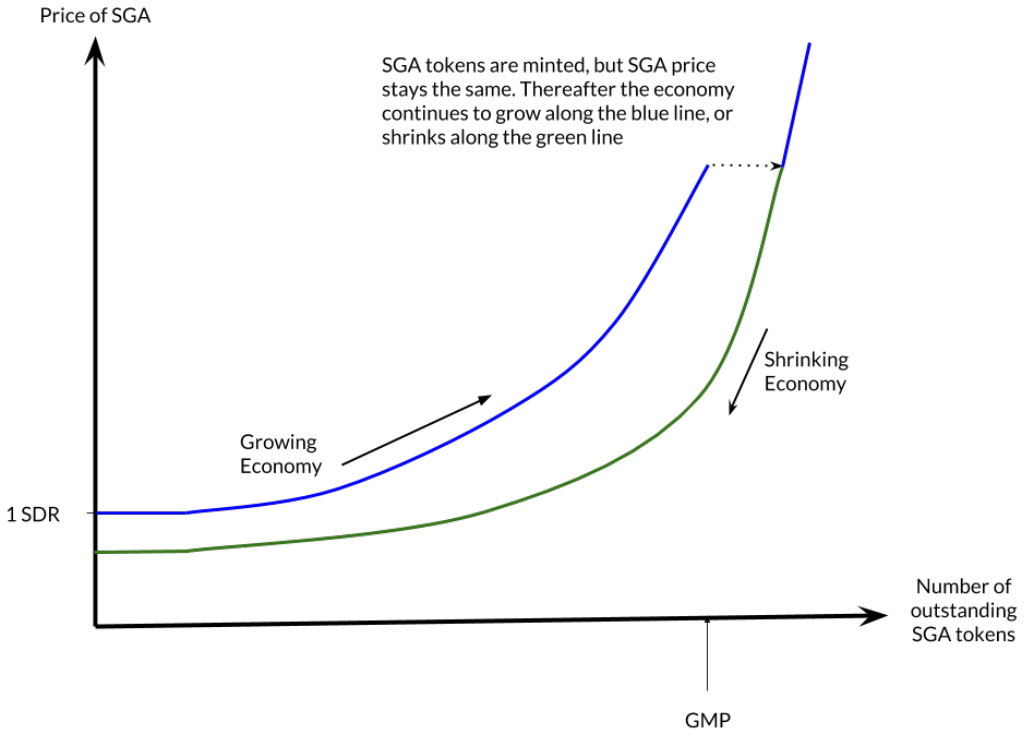


Figure 8: Price of SGR in several shrinking economy models, at various high-water marks (HWM) of Sögur's economy

The blue line indicates how the price of SGR increases when the economy grows. The orange line shows how the price of SGR drops if the economy begins to shrink having reached a high-water mark market cap of 1 billion SDR. And so on.

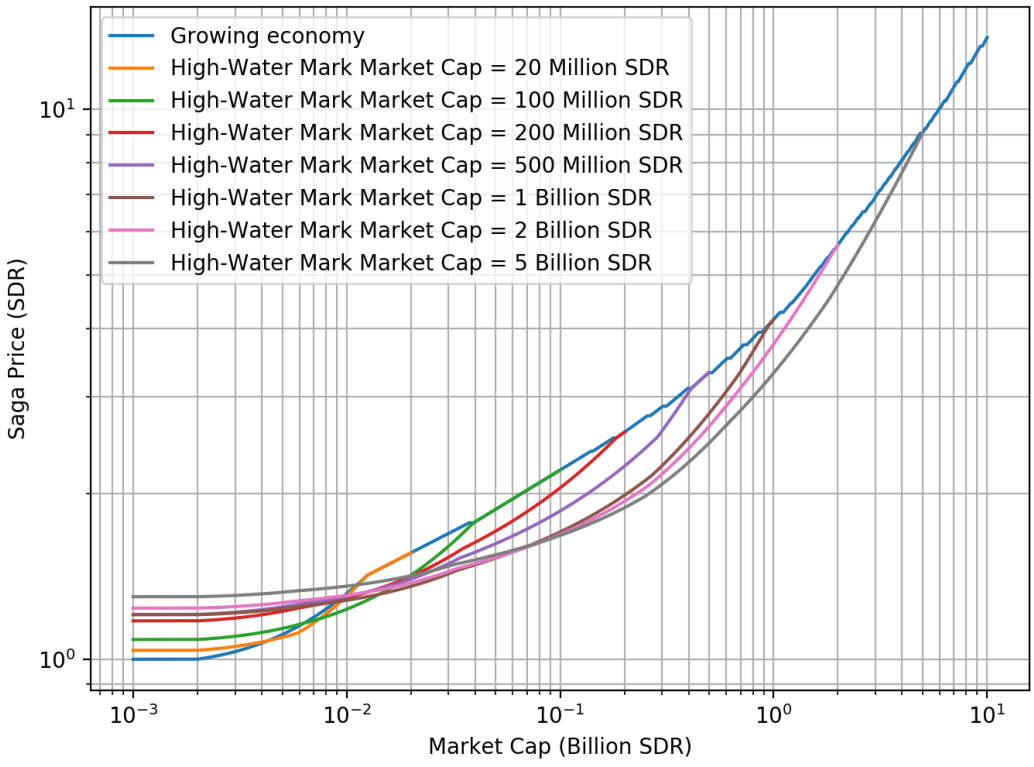
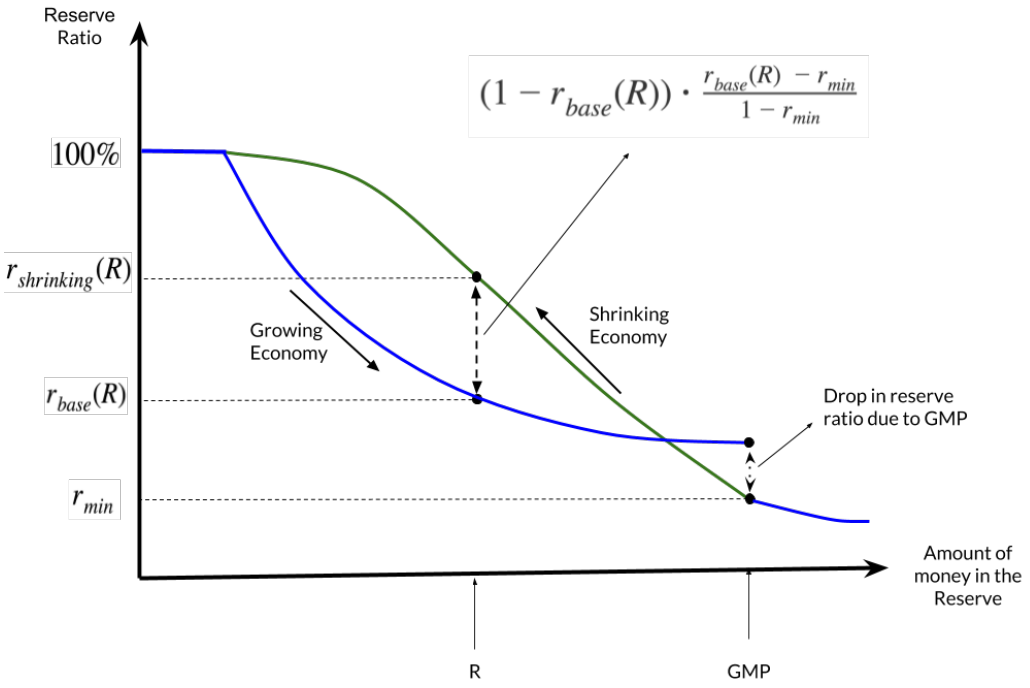


Figure 9: Illustration of Reserve Ratio around a GMP

When the economy shrinks, the reserve ratio is higher than it was when the economy grew. This counter-cyclical factor acts to stabilise the Sögur economy, discouraging any herding behaviour or a run on the bank scenario.



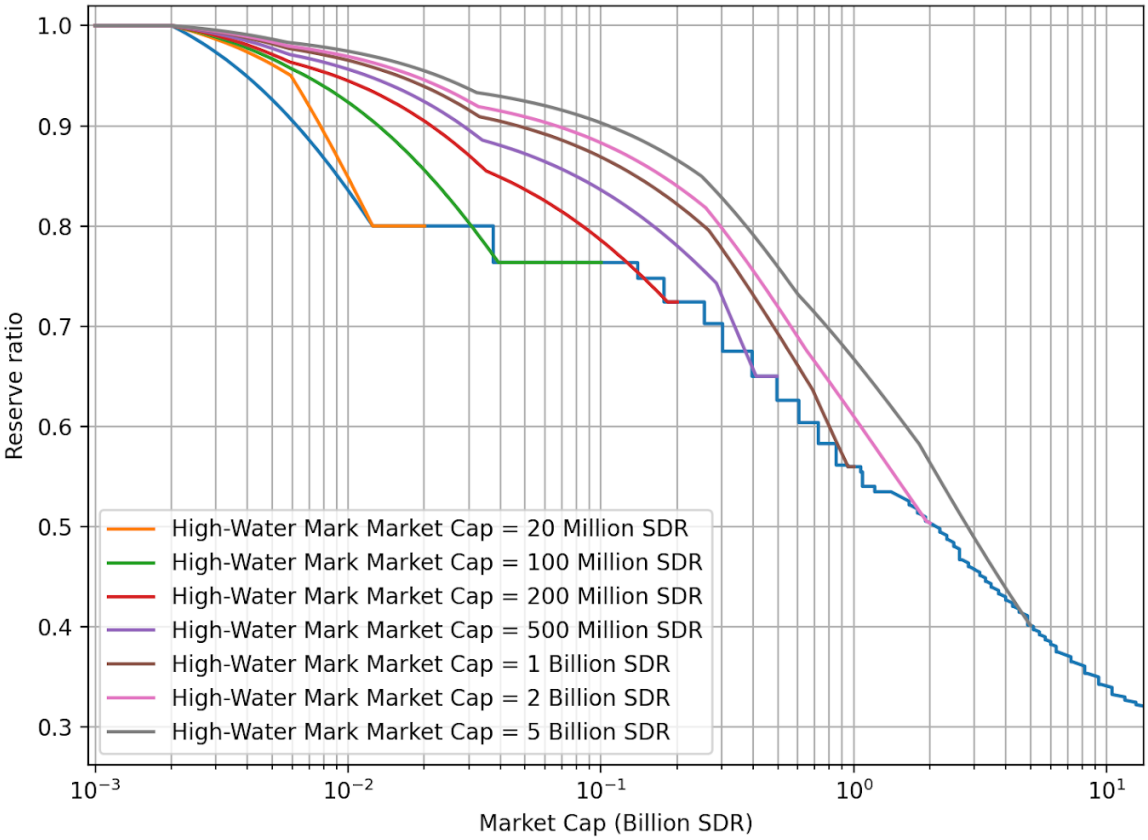


Figure 10: Reserve Ratio function of various shrinking economy models

9. PRICE ADJUSTMENT

Until now, we have described Sögur's model in its pure, theoretical setting, with little reference to how it is actually implemented. In the following chapters we describe our approach to translating the pure model into a framework that operates in the real world.

The issue we address here is the deviation of the Reserve value from the model's assumption. Sögur's model assumes the Reserve is strictly composed of the net proceeds from selling SGR. In practice, however, the Reserve would have other sources of income and expenses, not taken into account by the model.

Primarily, these include:

1. Accrued interest. The Reserve will be held in major regulated banks in liquid, low-risk assets that pay interest. The accrued interest is added to the Reserve.
2. Reserve management costs. Direct costs of holding the Reserve — e.g. bank commissions and blockchain transaction costs — are paid from the Reserve itself.
3. Revenue from the bid/ask price band. See next Chapter.
4. Changes to the value of ETH in the Liquidity Buffer. See chapter 11.

Therefore, at any given moment, there is a likely to be a divergence between what our model thinks the value of the Reserve is, and what the value of the Reserve actually is. We denote these two values as R_{model} and R_{actual} respectively.

Pricing Sögur should be based on the *actual* value of the Reserve. For example, if the Reserve accrues interest, the price of SGR should rise accordingly so owners of SGR benefit from the 'time value of money' — otherwise Sögur's usefulness as a store of value would be compromised.

Therefore, when implementing the Sögur model, the price employed by the Contract is the price given by the model, multiplied by the ratio between R_{model} and R_{actual}

In other words, $P_{actual} = \frac{R_{actual}}{R_{model}} \cdot P_{model}$

Note the reserve ratio remains constant, since both SGR price and the Reserve have changed by the same factor.

10. PRICE BAND

Introduction & Motivation

We have described how we built a model where the Contract determines SGR value by offering to buy and sell SGR at the same price.

However there are substantial advantages to allowing SGR price to fluctuate without the Contract always intervening.

In practice therefore, Sögur's Contract does not fully determine a single price for SGR, but rather determines a price band. The Contract offers to buy and sell SGR at different prices: it sells SGR at a price, P_{ask} , and buys back SGR at a lower price, P_{bid} .

There are several benefits to this mechanism:

1. Opening SGR to the free market:

If the Sögur Contract bought and sold SGR at the same price, most trading activity would happen with the Contract. As market demand for SGR rises or falls, participants would approach the Contract to buy or sell SGR, causing a rise or fall in the supply of SGR.

However, it makes no sense that the supply of SGR should rise and fall with every whim of the market.

Applying the price band framework, the supply of SGR does not change with every change in SGR holdings. Instead, Sögur's model defines a range in which it is deemed acceptable for SGR price to vary. SGR is free to rise and fall within this band according to market forces. Only when SGR price moves significantly, deviating from the band, will the Sögur Contract step in and take measures to slow down price movements.

2. Reducing operational costs:

In a model without a price band, most buying and selling of SGR occurs with the Contract.

However, in order to change the supply of SGR, the Contract must perform a currency conversion — since Sögur keeps its Reserves denominated in SDR while payments are received in other forms.

Performing currency conversions each time there is a fluctuation in market demand for SGR is expensive and cumbersome. These costs must be borne by someone; either by market participants, or by the Reserve — in which case it is borne by all SGR holders via the price adjustment mechanism described in the previous chapter.

On the other hand, with a price band framework, most trading activity will not involve the Contract, as commercial exchanges are likely to offer smaller bid/ask spread. In preference, the secondary market provides the majority of SGR liquidity, drastically reducing the number of currency conversions.

3. The Reserve accrues income from the price band:

Consider a scenario in which the market price of SGR rises to the top of the price band, before falling to the bottom. When the market price was at the top of the band, market participants bought SGR from the Contract at price P_{ask}. When market price was at the bottom of the band, market participants sold SGR back to the Contract at the lower price P_{bid}. The Sögur Reserve thus gained the difference, P_{ask} - P_{bid}, on every SGR token bought and then sold.

Why is this positive?

Recall that all profits of the Reserve fund are returned to SGR holders through an increase in the value of their SGR, as described in the previous chapter.

This is consistent with our general approach. Throughout this paper we stressed the wish to protect SGR holders against volatility in SGR price. The price band mechanism means that — if volatility does develop — market players pay the cost of the bid/ask spread, and long-term SGR holders are compensated for the volatility through an increase in the value of their SGR.

4. Finally, the price band gives an added layer of protection against market manipulations, such as front-running by blockchain miners:

Indeed, any strategy that attempts to make money buying and selling SGR tokens through the Contract is immediately weakened by the bid/ask spread.

Consequences of the Price Band

We have explained our motivations for including two separate prices in Sögur's model — the ask price and the bid price.

The material presented in previous chapters still applies; the pricing model from previous chapters determines the mid price of SGR - the average of the Contract's ask and bid prices.

In reality, SGR price in secondary markets can fluctuate within a price band around this value. The extent to which price can fluctuate before the Contract steps in to limit price movement is determined by the width of the price band.

Price Band Width

We define the width, w, of the Contract's price band to be the distance between the ask price and the mid price, given as a percentage of the mid price.

Under this definition, the Contract's ask price is $P_{ask} = P_{mid} \cdot (1 + w)$; and the bid price is $P_{bid} = P_{mid} \cdot (1 - w)$

For example, if the model's mid price is 100 SDR and the width of the price band is ±7%, then the Contract's ask price is 107 SDR and the bid price is 93 SDR.

The width of the price band is a measure of how much SGR price is allowed to fluctuate in the market before the Contract intercedes.

We had several considerations for determining the width of the price band.

- When Sögur's economy is small the width should be minimal: just enough to cover operational costs for trading with the Contract.

When Sögur's economy is in the earliest stages of its development we want the Contract to fully control the price of SGR. Moreover, secondary market liquidity is likely to be limited. We determined the Contract should employ a price band of width ±0.15% at this stage, on par with fees charged by major exchanges.

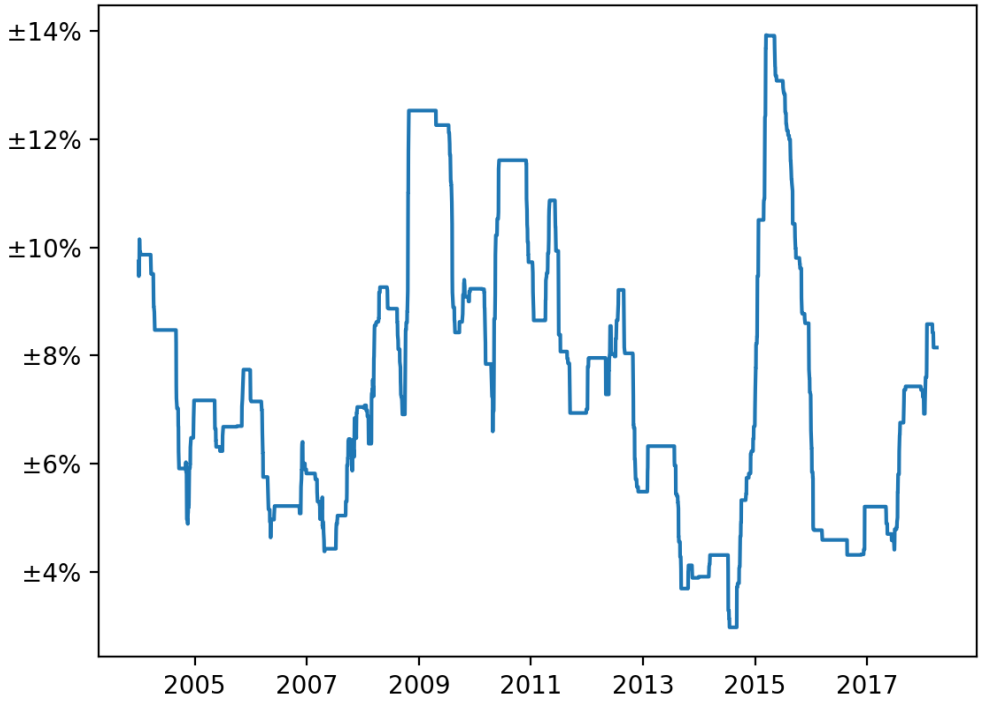
- The width of the price band should increase as Sögur's economy grows. As the market cap of SGR grows we wish to give the secondary market a greater role in determining price.

- The maximum value of the price band width, when Sögur's economy has reached first-rank prominence, should allow SGR price fluctuations similar to those of major fiat currencies. We chose a value of ±15%. This is supported by two observations:

    A. A criterion for countries to join the Eurozone is that their currency remains within a 15% band around a targeted Euro exchange rate value for at least two years.[5]

    B. Having studied exchange rates of several major currencies, we found that 1-year price fluctuations lay within a 15% band even in times of exceptional crisis (e.g. 2008 financial crisis).

Figure 11: Width of fluctuations in EUR-USD Exchange Rate over a rolling 250 market day period



[5] https://ec.europa.eu/info/business-economy-euro/euro-area/enlargement-euro-area/introducing-euro/adoption-fixed-euro-conversion-rate/erm-ii-eus-exchange-rate-mechanism_en

Price Band Width Framework

We modelled the width of the price band to grow inversely to the reserve ratio; the more the market gives trust to Sögur's economy (i.e. reserve ratio is lower), the greater the market role in determining SGR price (i.e. price band is wider).

Opening up SGR price discovery to secondary markets requires these markets be both active and liquid. The model maintains a minimal bid-ask spread, until this is the case.

The width of the price band in Sögur's model is defined as follows:

$$w = \frac{c_1}{r} + c_2$$

Where r refers to the value of the reserve ratio and c_1 and c_2 are constants that sets the width of the price band to be 0.15% when the reserve ratio is 100% and that the width reaches its maximum value of 15% when the reserve ratio reaches its minimum value of 10%.

Table 3: Price-Band Width at various points in Sögur's model

Market Cap	Price-Band Width	Reserve Ratio
(SDR)	(%)	%
1M	0.15%	100.00%
3M	0.20%	97.33%
10M	0.48%	83.54%
30M	0.56%	80.00%
100M	0.66%	76.35%
300M	0.85%	70.25%
1B	1.45%	55.97%
3B	2.11%	45.73%
10B	3.34%	34.08%
30B	4.10%	29.44%
100 B	5.54%	23.45%
300 B	6.70%	20.12%
1 T	10.02%	14.32%
> 3 T	15.00%	10.00%

Figure 12: Width of Price Band in Sögur's model

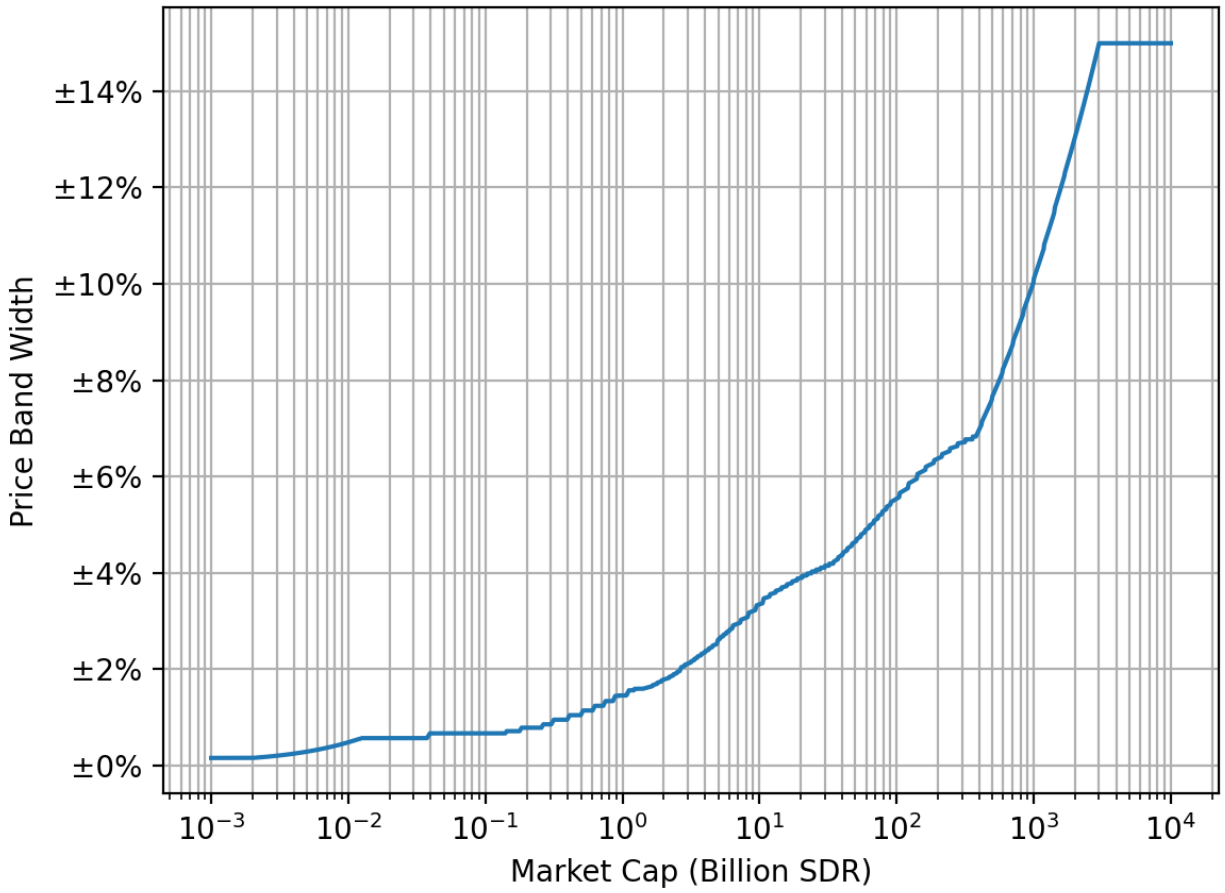


Figure 13: Bid and Ask Prices of Sögur's model

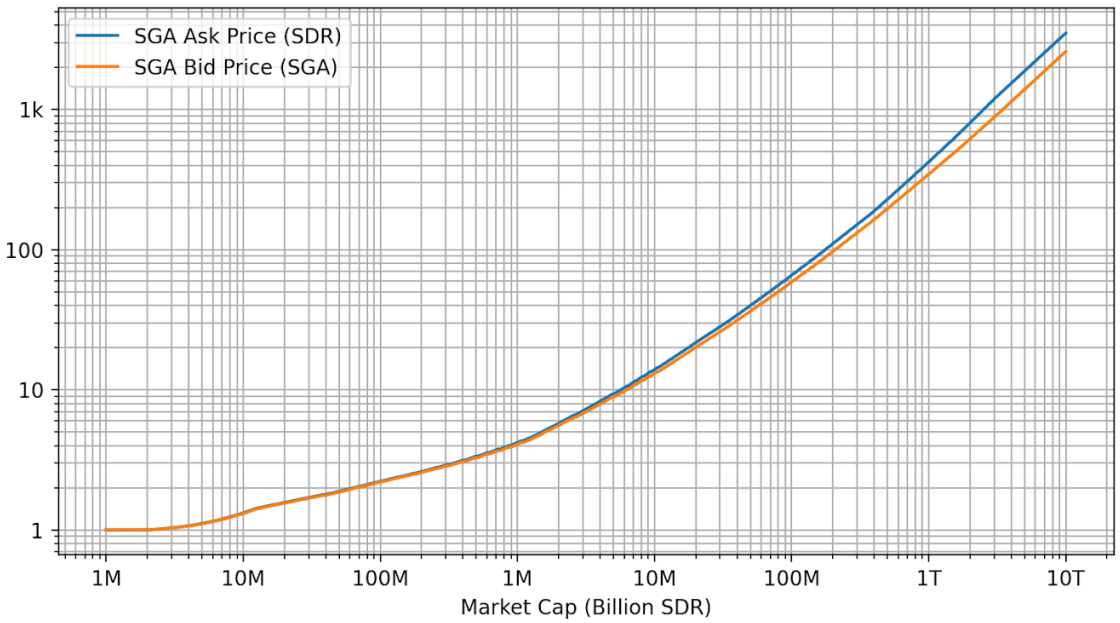
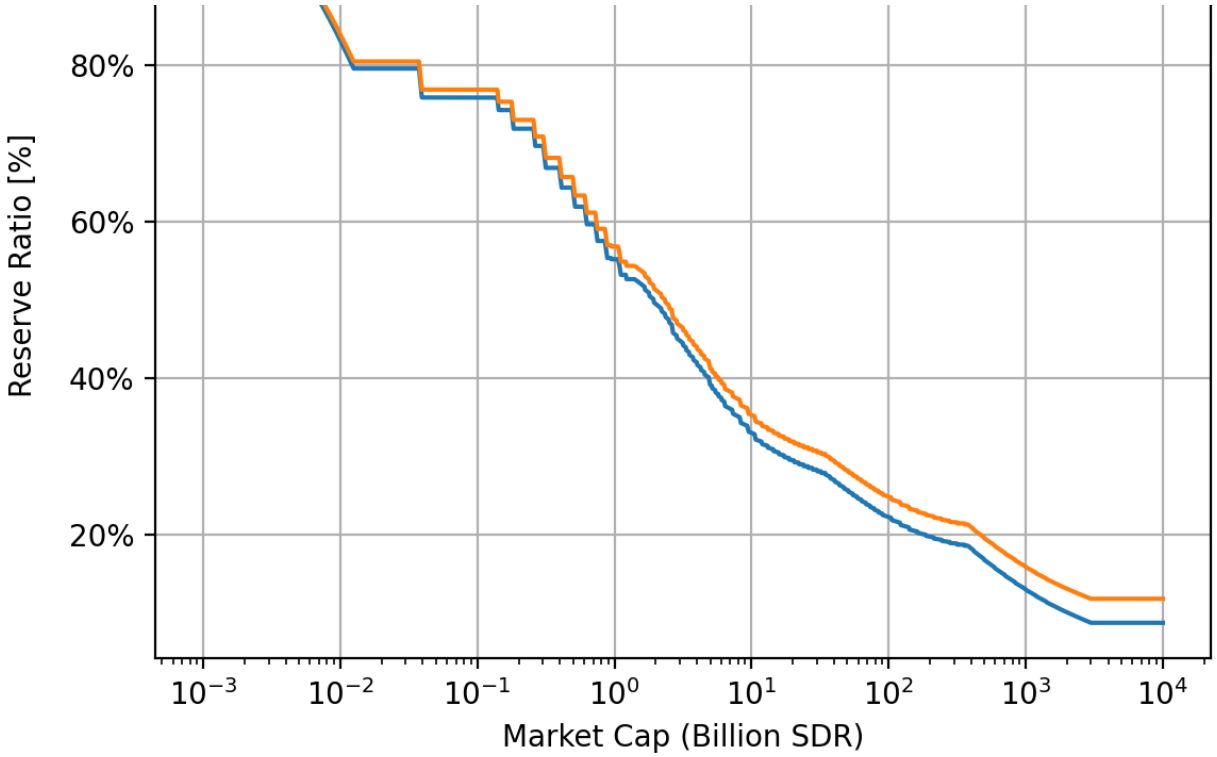


Figure 14: Reserve ratio band



11. OPERATING ON THE BLOCKCHAIN

Working with ETH

We implement code-based elements of our monetary model as a smart contract on the Ethereum blockchain network, as opposed to running on private servers. The crucial and sensitive nature of the topics involved — the ability for the market to control SGR supply, and the relation between SGR supply and SGR price — calls for a decentralised implementation. Decentralisation guarantees complete transparency, assuring holders of SGR that our model is actually carried out as described, and that there is no private body that has the ability to manipulate or change the model.

Our model uses the IMF's SDR as its unit of account, but there is no way of sending SDR over the Ethereum blockchain at this time. Indeed, there is no way of sending any fiat currency over any blockchain.

Consequently, when implementing Sögur's liquidity provision model, money is transferred to and from the Contract in the form of ETH, Ethereum's native currency. Participants that wish to buy SGR send ETH to the Contract; and participants that sell SGR back to the Contract are reimbursed in ETH.

The model still operates based on the SDR: when participants send ETH to the Contract to buy SGR, the Contract calculates the value of the ETH in SDR[6] and generates SGR accordingly; when participants sell SGR, the Contract calculates how much SDR the seller should receive and sends the equivalent amount[7] in ETH.

Balancing Liquidity & Risk

In order to provide immediate liquidity to sellers of SGR, the Contract keeps an amount of ETH on hand in a *liquidity buffer*. All other ETH received by the Contract is converted into fiat, and deposited in Sögur's Reserve.

When participants sell SGR to the Contract, ETH is withdrawn from the liquidity buffer to reimburse the seller. If ETH levels in the liquidity buffer fall below a defined threshold, fiat is withdrawn from the Reserve, converted into ETH, and sent to replenish the liquidity buffer.

There is an inherent tradeoff here: in order to provide immediate liquidity, ETH must be kept in the liquidity buffer. On the other hand, keeping part of the Reserve in ETH exposes the Reserve — and thus SGR price — to fluctuations in ETH price.

We intend to keep Sögur's liquidity buffer at a level sufficient for the Contract to provide liquidity under 'normal' market conditions, while the majority of the Reserve will be kept in fiat currencies which replicate the currency composition of the SDR. In addition, to ensure liquidity, reduce transaction costs and simplify operations, some of the reserve may be held in licensed EMIs and trustworthy stablecoins

During times of high volume of selling SGR, the liquidity buffer may be depleted faster than we can top it up. The Ethereum network, and consequently Sögur's smart contract, works 24-7, while withdrawals from bank accounts are limited to banks' business hours.

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[6] According to the ETH/SDR exchange rate offered to Sögur by its currency exchange providers

[7] According to the SDR/ETH exchange rate offered to Sögur by its currency exchange providers

To mitigate some of this gap, we work with a number of liquidity providers that will allow us to acquire ETH even during off-hours. However, the amount these providers can supply without further deposits from banks is limited.

Under exceptional market conditions, it is possible that participants will want to sell SGR to the Contract and the liquidity buffer will not have enough ETH to reimburse them immediately. In such cases, participants may still sell SGR back to the Contract, and will enter a queue to be reimbursed once the liquidity buffer has been topped up. Reimbursement of ETH is done on a first-in-first-out basis.

The queue stores obligations in terms of SDR, not ETH. When the liquidity buffer is topped up, ETH is sent to participants in the reimbursement queue according to the current SDR-ETH exchange rate. Thus, whilst waiting for reimbursement, participants are not exposed to the value of ETH nor to the value of SGR, but only to our unit of account, the SDR.

The introduction of an ETH redemption queue clearly impairs the ability of SGR holders to sell SGR immediately, and therefore diminishes Sögur's promise of liquidity. However, we consider this mechanism to be best-possible under the circumstances, as it mitigates the other risk of exposure to ETH value fluctuations.

Protection Against Front-Running

A consequence of operating on a public blockchain is that requests to trade SGR with the Contract are publicly visible, even before they are executed. Moreover, orders are not necessarily executed in the order they are made. This exposes Sögur to the problem of *front-running*.

Since our model sets the price of SGR according to the number of SGRs in circulation, buy transactions result in an increase in SGR price, while sell transactions result in a decrease in SGR price. If a large transaction request is made, other players could capitalise on the knowledge about future price changes by submitting a similar request that they hope will be carried out before. If they are successful, after the original order transaction is executed, attackers can close their position with an assured gain.

The result is that the original participant receives a less favourable price than otherwise.

Front-running can be done by blockchain miners or non-miners alike, and is a problem that affects many blockchain-based projects.

Our price band provides some protection against front-running. Given that participants buy SGR at the model's ask price and sell SGR at the lower, bid price, a front-run will only be profitable against orders that are large enough to cause a change in SGR price wider than the price band.

For example, an order to buy SGR will only be susceptible to a front-running attack if it is large enough to cause the Contract's bid price to rise above the previous ask price.

The fact that we designed the width of Sögur's price band inversely to the reserve ratio means that SGR participants are always protected to some extent from front-running: either by a high reserve ratio or by a wide price band:

When the SGR economy is small, the price band is narrow. But the reserve ratio is high. So SGR price is resistant to change, and large transactions are needed to move price through the price band.

When the economy is larger, the reserve is low and SGR price is more susceptible to change. But this is precisely the stage when the price band is wide, so large transactions are still needed to create a price change larger than the width of the price band.

The following table gives the sizes of transaction that make front-running attacks profitable at various stages in Sögur's economy.

Table 4: Susceptibility to Front-Running at various points in Sögur's model[8]

Market Cap	Price-Band Width	Reserve Ratio	Size of Order Susceptible to Gaming	
(SDR)	(%)	%	(SDR)	(% of Reserve)
10M	±0.47	83.5%	0.2M	2.5%
30M	±0.56%	80.0%	1.4M	5.8%
100M	±0.66%	76%	4.4 M	5.79%
300M	±0.84%	70.2%	12.2 M	5.79%
1 B	±1.45%	55.9%	38.9 M	6.96%
3 B	±2.11%	45.7%	90 M	6.56%
10 B	±3.34%	34.0%	333 M	9.77%
30 B	±4.1%	29.4%	1.13 B	12.9%
100 B	±5.54%	23.4%	37 B	14.37%

[8] When SGR market cap is below 1B SDR, the price band remains constant though the reserve ratio decreases. Price is more amenable to change: a smaller percentage of the overall reserve is required to move SGR price across the price band.

12. STANDALONE ECONOMY

Sögur's reserve-based model sets the price of SGR in terms of our Reserve numeraire — the SDR. This means SGR's purchasing power is related to the purchasing power of SDR: if SDR weakens or strengthens relative to some other unit of account, then the Reserve-backed value of SGR weakens or strengthens in a similar manner.

While Sögur's economy is developing, basing its value on a Reserve denominated in SDR is beneficial; the currencies that make up the SDR are established and widely trusted units of account on which to base our nascent currency.

However, at some point, if Sögur's economy continues to grow, it will no longer make sense to value SGR in terms of SDR. If Sögur's market cap reaches into the trillions, Sögur itself would be highly established and no longer require support from the SDR. At this stage, Sögur can take a 'life of its own' where it is not directly affected by rises and falls in the value of the Reserve.

Hence, at some stage, the reserve-based pricing model should be dropped in favour of a different model.

Setting the behaviour of a global digital currency of first rank prominence, when we are so far from this, is an impossible task. We do not presume to describe a world so different from today. Instead, we present a 'default' behaviour for how Sögur's currency will behave should it reach this stage. The default behaviour can be revised by the community[9] if it turns out to be sub-optimal.

This default behaviour is to set a hard cap on the supply of SGR tokens.

Above this cap, the Contract does not mint new SGR tokens. However SGR tokens can always be sold back to the Contract under the original pricing model. If this occurs, the Contract will continue to re-issue SGR tokens until the hard cap is once again reached.

Effectively, the Sögur Contract ceases to be a liquidity provider for both buying and selling SGR tokens and only offers to buy SGR from the market. Thus, the Contract sets a floor for the price of SGR in terms of the SDR, but not a ceiling.

We chose the hard cap to occur when Sögur's market cap reaches a very high value to push it far into the future -5 trillion SDR (~7 trillion USD). At this stage the model's bid price is 1390 SDR.

From this point onwards, SGR price can increase freely in the secondary market, effectively detaching SGR value from the Reserve.

The Contract still provides a floor for the price of SGR. If the price of SGR in secondary markets ever falls below 1390 SDR, participants will sell SGR back to the Contract. The original pricing model will then kick back into action.

Thus, once the hard cap has been reached, SGR price is still supported by its Reserve but is no longer constrained by it.

[9] Sögur has established a research institute with the goal of laying the framework for such future decision making.

APPENDIX A: THE PARAMETERS USED TO BUILD SÖGUR'S MODEL

In this paper, we detailed our aims and motivations in designing Sögur's monetary model, and the conclusions we reached. There were four main frameworks we had to design:

1. The behaviour of the reserve ratio function, for a growing economy
2. Saga Genesis Minting Points
3. The reserve ratio in a shrinking economy
4. The width of the price band

Details of the last two frameworks — the shrinking economy and the price band width — were already given in full in the paper above.

Below, we provide the full details of exactly how we built the first two frameworks — the reserve ratio function for a growing economy, and Genesis Minting points — and the parameters used.

Note the model we describe here determines the mid price of the Contract's ask and bid prices — the ask and bid prices themselves are determined by the mid price and the width of the price band.

1. The Reserve Ratio Function

The general framework for Sögur's reserve ratio model is:

- The reserve ratio should remain at 100% until a certain threshold in market cap is reached.
- Until this threshold, SGR price remains fixed at 1 SDR.
- Afterwards, the reserve ratio decreases piecewise linearly in accordance with the number of SGR in circulation.
- The reserve ratio decreases to a minimum of 10%, then stays at 10% thereafter.

We consider the amount of money in the Reserve as the best measure of Sögur's success, so we designed the model using that as the main independent variable.

We built the reserve ratio function as follows:
There were several points we wished the function to go through. We call these points 'target points'. More precisely, a target point is a target of the form: "We would like the reserve ratio to be $x\%$ when the Reserve reaches a value of y SDR".

For example, the first target point was: "The reserve ratio should stay at 100% until 2M SDR has been put in the Reserve."

The next target was: "The reserve ratio should reach 80% when the Reserve value reaches 10M SDR."

In the interval between these two target points, the reserve ratio decreases linearly from 100% to 80%. We calculate exactly what the rate of decline should be so that, if nothing changes, the value of the Reserve will be 10M SDR when the reserve ratio reaches 80%. In other words, we calculate exactly what the slope of the reserve ratio function should be, so that we hit our second target exactly.

The only complication: when a Genesis Minting Point occurs, the reserve ratio drops artificially due to the minting of new SGR tokens — i.e. the reserve ratio function is momentarily knocked off course. Therefore, we re-calibrate the slope of the reserve ratio function to continue to aim for the next target point.

Target points used for our model:

Target Point No.	0	1	2	3	4	5	6	7
Reserve Size (SDR)	0	2 M	10 M	100 M	750 M	10 B	75 B	300 B
Reserve Ratio	100%	100%	80%	80%	75%	30%	20%	10%
Market Cap (SDR)	0	2 M	12.5 M	125 M	1 B	33 B	375 B	3 T

The calculation to find the slope of the reserve ratio in a particular interval, i, is as follows:

Let us denote the number of SGR in circulation, the reserve ratio, and the value of the Reserve at the beginning of the interval as N_0, r_0 and R_0 respectively.
Let the next target be that the reserve ratio should equal r_1 when the Reserve value equals R_1.
Let N_1 be the number of SGR tokens in circulation when the target point is achieved at the end of the interval.
Let α_i, β_i be the parameters that determine the shape of the reserve ratio function in interval i — so the reserve ratio during the interval is given by $r = \alpha_i - \beta_i N$
Note that N_1, α_i, and β_i are not yet known. We can find their values by solving the following three equations:

1. $r_0 = \alpha_i - \beta_i N_0$ (reserve ratio is r_0 at the beginning of the interval)
2. $r_1 = \alpha_i - \beta_i N_1$ (reserve ratio is r_1 at the end of the interval)
3. $R1 = R_0 \cdot \left(\frac{N_1 \cdot r_0}{r_1 \cdot N_0} \right)^{1/\alpha_i}$ (Reserve amount is R_1 at the end of the interval[10])

It is possible — due to drops in the reserve ratio that occur at Genesis Minting Points — that the value of the reserve ratio could be slightly *below* the next target value. In other words, it is possible for the reserve ratio to decrease below the target value r_1 before the Reserve value reaches R_1. In this case we keep the reserve ratio constant until the Reserve value reaches R_1.

[10] Using the equation for R as a function of N that was derived in the main text (note that in the main text, the constants R0, N0 and r0 were all subsumed into the constant omega).

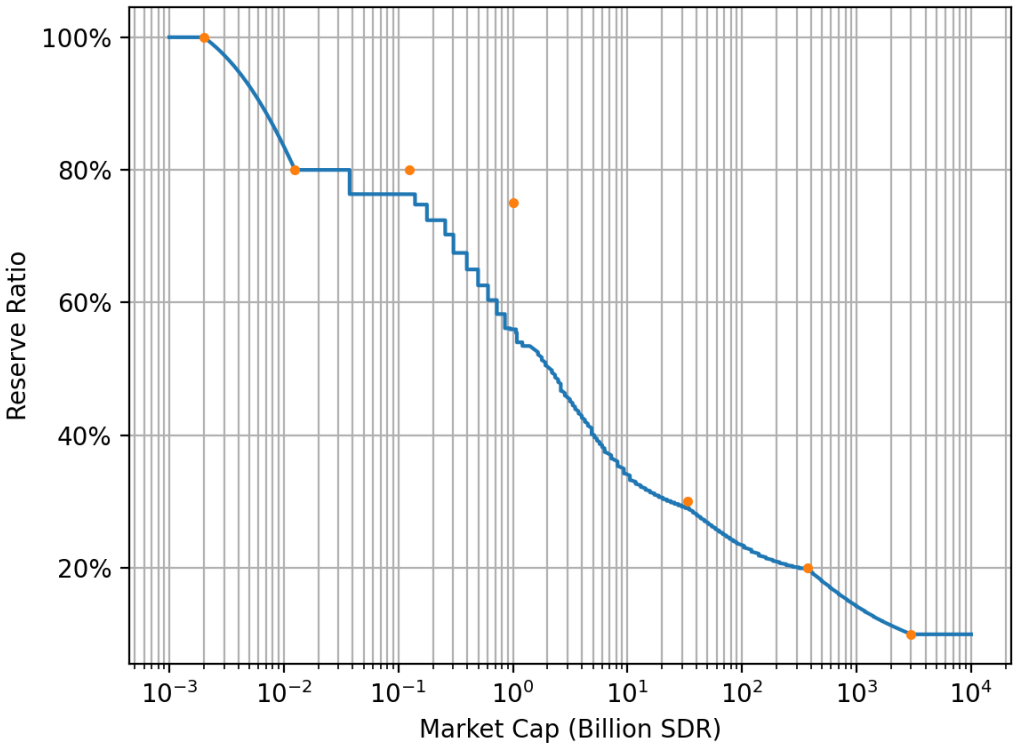

Figure A.1: Reserve ratio as a function of the Reserve value, with the model's target points superimposed in orange

2. Saga Genesis Minting Points

Saga Genesis Minting Points are where the Contract mints SGR tokens for SGN holders, with no increase to the Reserve.

The general framework and aims that we detailed above were:

- Overall, 7 SGR tokens should be minted for each SGN token.[11]
- The minting of these tokens should be done step-by-step as the economy grows.
- The number of tokens minted at each Minting Point should be 'reasonable'; it should balance the aims of providing sufficient return for SGN holders, with the requirement that impact to Sögur's pricing model be minimised.
- We should ensure that SGR tokens minted for SGN holders do not represent a significant proportion of the total SGR economy.

There were two aspects we needed to design:

A. The location of each Genesis Minting Point
B. The number of SGR tokens generated at each Minting Point

[11] Note: there are 107 million SGN tokens in total.

In September 2020, Sögur's monetary model was revised - see Appendix E for more details on the changes in the revised model. Under the revised model, the SGN minting parameters were amended to ensure that the SDR value of each SGN as a function of Sögur's market cap remains the same as in the original model. Therefore, the parameters below refer to how the original model's SGN minting framework was built, as it serves as the basis of constructing the revised SGN minting framework.

A. Location of each Genesis Minting Point

Recall that if a Minting Point occurs when there are N SGR tokens in circulation, and if n new tokens are minted, then the reserve ratio decreases by a factor of $(1 - \frac{n}{N+n})$

The fraction $\frac{n}{N+n}$ is a measure of the extent to which a Minting Point impacts Sögur's economy.

In general, the later we can put off Minting Points the better — since if $N$is larger, the effect to the economy will be smaller. Therefore, we aimed to have as few minting points as possible, while also acknowledging the need to provide an adequate rate of return for SGN holders.

We set the first Genesis Minting Point to occur when the Reserve value reaches 25M SDR.

When designing the location of the other Genesis Minting Points we used two parameters: *abs_step* and *rel_step*.

If the previous Genesis Minting Point occurred when the Reserve value was R_0 SDR, then the next minting point occurs once the Reserve has increased by *abs_step* + $R_0 \cdot$ *rel_step* SDR.

We used these parameters to ensure that, in order to reach the next Minting Point, Sögur's economy must grow significantly — both in relative and in absolute terms.

B. Number of SGR tokens generated at each Minting Point

Once we have decided the location of a Minting Point, we need to decide how many SGR to mint.

In addition to our aim of having a balanced rate of return for SGN holders, we want to make sure that SGN holders do not control a significant proportion of the SGR economy. Therefore, we mint SGR tokens in reference to the number of SGR tokens already in circulation.

For this we used a parameter, *minting_factor*.

If there are M SGR tokens in circulation at a Minting Point - not counting SGR tokens that were themselves minted for SGN holders — then we mint a further $M \cdot$ *minting_factor* tokens.

The *minting_factor* parameter helps control the percentage of Sögur's economy that SGN holders own — so that early backers do not swamp Sögur's economy. In Sögur's model, at their maximum, SGN holders are entitled to less than 30% of all SGR tokens.

Parameters

We built our model so that the minting of SGR tokens for SGN holders happens at different rates.

In the first stage we aim to get the conversion ratio of SGN-SGR to equal 1 by the time that the Reserve value is 500 million SDR. For this stage we do not use the minting_factor parameter. Instead the amount of tokens minted depends on the Minting Point's distance to the target. In subsequent stages, the rate of minting SGR tokens is steadily reduced.

The parameters we used for each stage are:

#	Up to conversion ratio	abs_step	rel_step	minting_factor
1	1	50M SDR	1%	n/a
2	4	50M SDR	1.5%	1%
3	10	150M SDR	5%	1%
4	15	250M SDR	12%	1%

A table of all Genesis Minting Points is given in Appendix D.

Vesting Points

70% of all SGN tokens are created before the launch of the Sögur economy. The rest, belonging to Sogur Monetary Technologies, are minted in three steps (10% at each step), when the economy reaches certain thresholds (Vesting Points). Delaying minting of SGN allows our model to mint fewer SGR tokens in the early stages of Sögur's economy - when it is most affected by minting.

When a Vesting Point is reached, new SGN tokens are minted along with SGR tokens needed to cover minting points already passed. These Vesting Points are like regular Minting Points, but here the minted SGR are used to cover the new SGN and do not increase the SGN-to-SGR conversion ratio.

The three Vesting Points and their vesting condition are:

Vesting Point Number	1	2	3
Reserve Value at Vesting Point (SDR)	600 M	1.25 B	2 B
Market Cap at Vesting Point (SDR)	1.11 B	2.67 B	4.98 B
Percentage of total SGN minted at Vesting Point	10%	10%	10%

APPENDIX B: MONTE CARLO SIMULATIONS — SENSITIVITY TO VOLATILITY

In this paper we described our efforts to build a pricing model that *curtails* SGR price volatility.

We designed the reserve ratio to be high when Sögur's economy is small in order to reduce volatility: a large number of new SGR tokens must be purchased before SGR price moves significantly. Moreover, the price band is narrow, meaning the Contract steps in frequently to change SGR supply and temper SGR price volatility.

When Sögur's economy is larger, the reserve ratio is lower: a smaller relative change in Sögur's Reserve is needed for SGR price to move[12]. In addition, the price band is wider. Thus, as the SGR economy becomes more established, the Contract plays a smaller role in regulating SGR price, and increasingly allows market forces to determine SGR price.

Here we present the results of Monte Carlo simulations on Sögur's pricing model, that aim to gauge the sensitivity of SGR price to volatility at various stages in Sögur's development.

In our experiments, we simulated daily percentage changes in SGR market cap and then looked at how the price of SGR would be affected. We generated the percentage changes from a normal distribution with mean zero[13]. Each simulation had two input parameters: the starting value of the market cap of SGR, and a value for the standard deviation of the daily percentage changes in the market cap.

We generated market cap time series, consisting of 365 periods. The time series begin with Sögur at the given initial market cap, and SGR price lying in the middle of the Contract's bid/ask price range.

At each step, a percentage increase or decrease to the market cap is generated at random from the normal distribution. The corresponding change in SGR price is then calculated as follows:

We assume SGR price moves in tandem with the market cap with no change in the supply of SGR, for as long as SGR price remains within the Contract's bid/ask range. If price does leave this range, then we assume people buy or sell SGR tokens with the Contract until the market cap given by Sögur's model matches the new market cap.

Below are figures showing results from ten Monte Carlo simulations. We generated percentage changes in market cap randomly from a Normal(0, 1%) distribution and considered the effects of these changes at different points in Sögur's economy.

The results are consistent with our theoretical results from Chapter 5; the lower the reserve ratio, the more magnified the corresponding price changes were.

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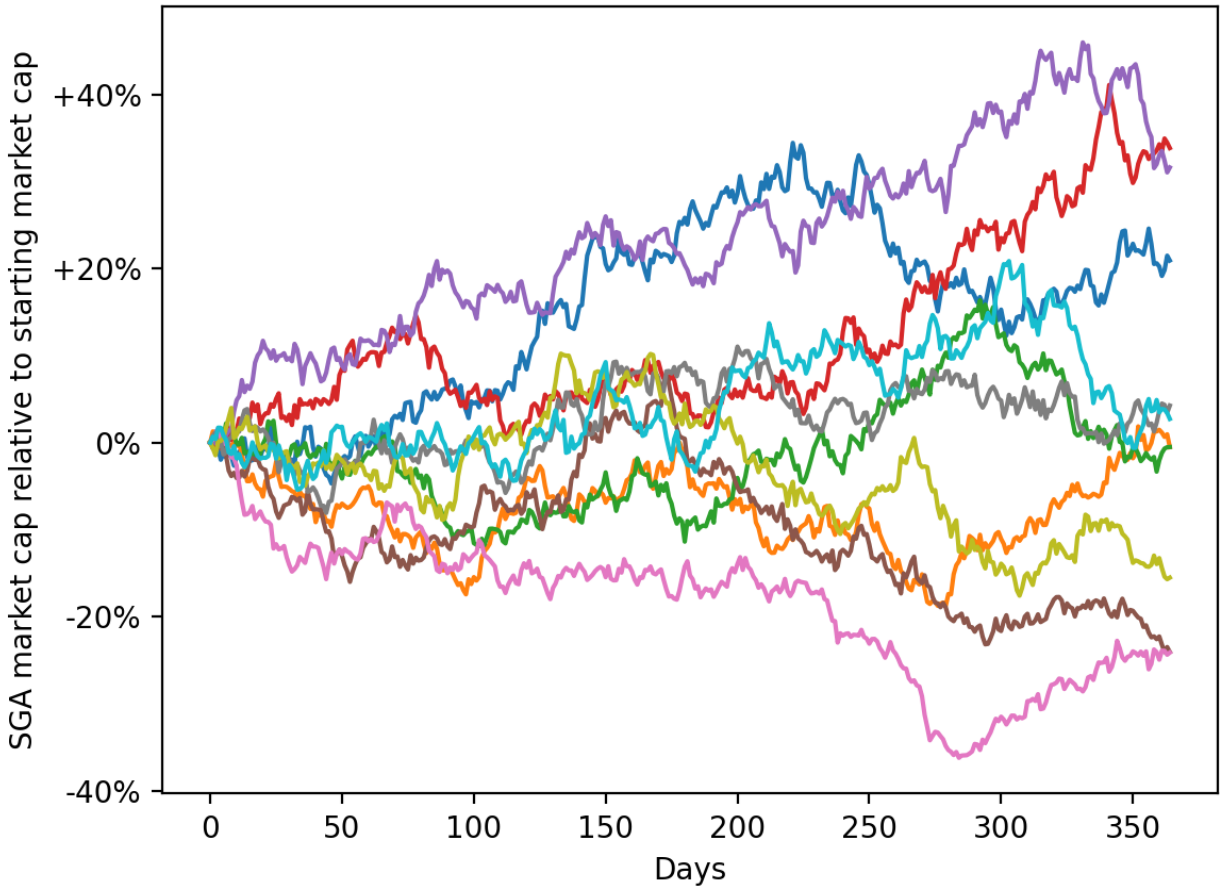
[12] In absolute terms this still amounts to a significant amount of cash flow, as the Reserve is much larger.

[13] Zero mean so that market cap is equally likely to increase or decrease. We note that considering a non-zero mean (drift) would not provide more insight.

Figure B.1: Ten random time series of fluctuations in SGR Market cap.
Standard deviation of the daily percentage changes is 1%



Figures B.2-7: The effect on SGR Price of these market cap fluctuations, at different stages in Sögur's economy. The original market cap fluctuations are shown in faded lines.

In Figure 2, the reserve ratio is 100% and fluctuations in the market cap have no effect on SGR price — except for within the Contract's narrow price band. In later figures, the reserve ratio is lower and the impact on SGR price is larger.

Figure B.2: Starting Market Cap 1M SDR. Reserve ratio 100%.
Width of Price Band ±0.15%

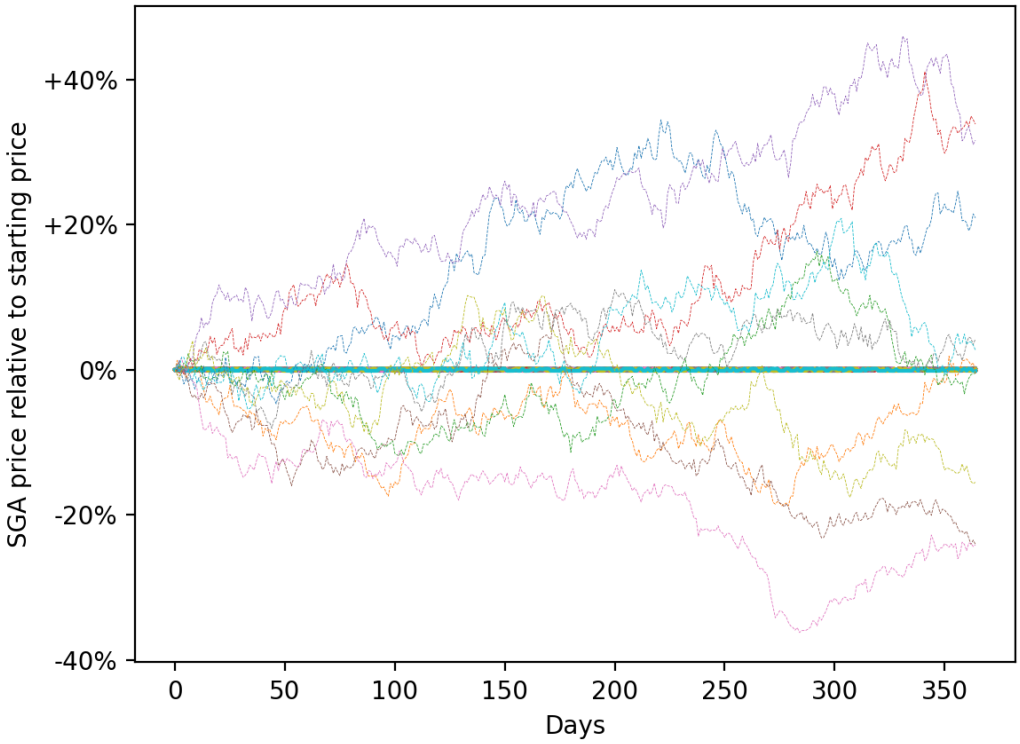


Figure B.3: Starting Market Cap 10M SDR. Reserve ratio 84%.
Width of Price Band ±0.47%

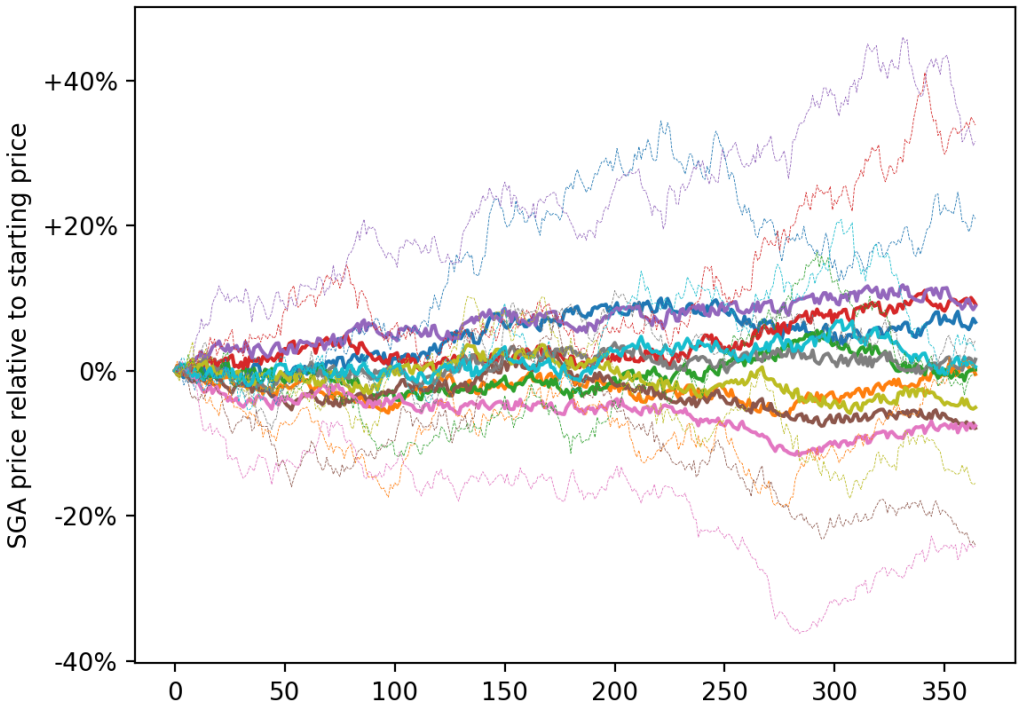


Figure B.4: Starting Market Cap 100M SDR. Reserve ratio 76%.
Width of Price Band ±0.66%

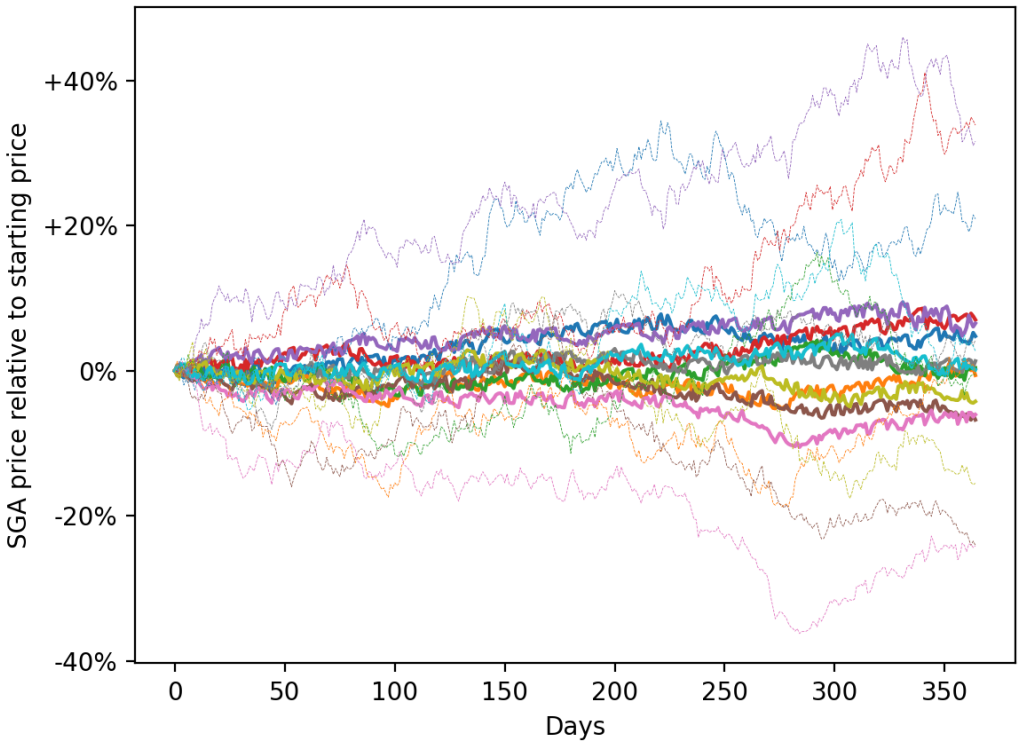


Figure B.5: Starting Market Cap 1B SDR. Reserve ratio56%.
Width of Price Band ±1.4%

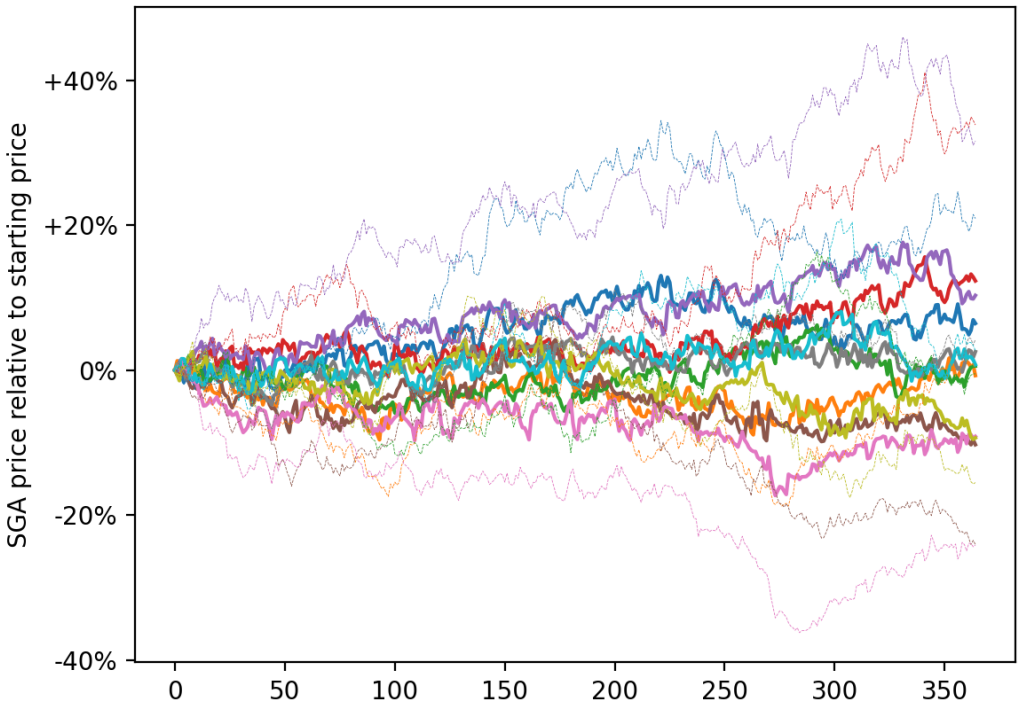


Figure B.6: Starting Market Cap 10B SDR. Reserve ratio 34%.
Width of Price Band ±3.4%

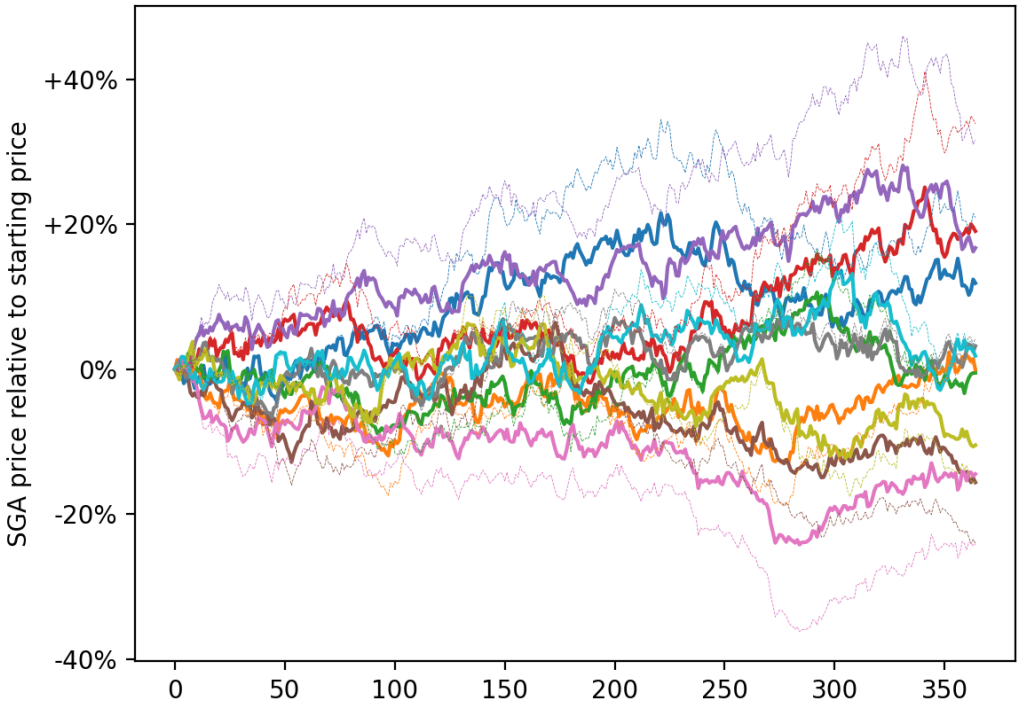
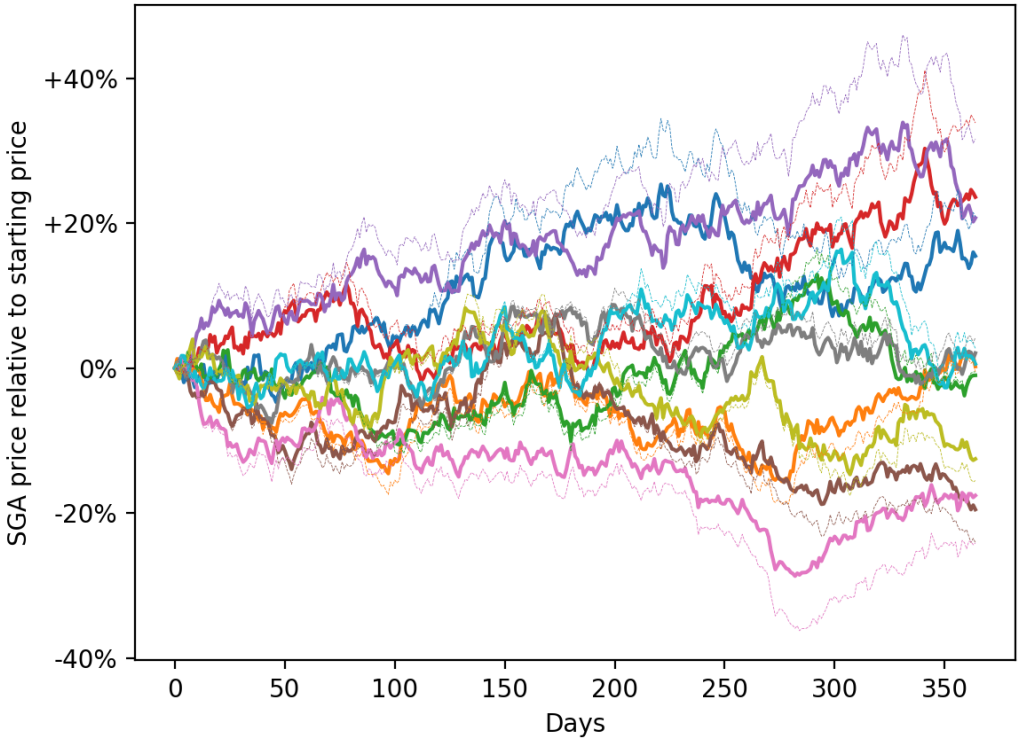


Figure B.7: Starting Market Cap 100B SDR. Reserve ratio 23%.
Width of Price Band ±5.5%



We now present quantitative results for further illustration.

For a given time series, we define its relative volatility to be its standard deviation divided by its mean. For example, if the average price of SGR across a time series is 5 SDR, and the standard deviation is 0.1 SDR, then the relative volatility is 0.1 / 5 = 0.02 (2%).

For a given pair of simulation parameters, we performed 20,000 Monte Carlo simulations and calculated the relative volatilities of the market cap time series and the SGR price time series. We then took the average of the volatilities across all 20,000 simulations.[14] These values give an indication of how sensitive SGR price is to volatility in its market cap at the given starting points.

Simulation Results - We present results for six different starting market cap values. For each market cap value, we considered standard deviations of 0.5% and 1% in the daily market cap percentage changes.

Simulation parameters:				Simulation results:	
Starting Market Cap	Starting Reserve Ratio	Starting Width of Price Band	Std of Daily Market Cap Percentage Changes	Market Cap Volatility	Price Volatility
(SDR)	(%)	(%)	(%)	(%)	(%)
1M	100%	±0.15%	0.5%	3.61%	0.14%
			1%	7.24%	0.14%
10 M	83.5%	±0.47%	0.5%	3.61%	1.2%
			1%	7.24%	2.3%
100 M	76.3%	±0.66%	0.5%	3.61%	1.00%
			1%	7.24%	1.8%
1 B	56.0%	±1.44%	0.5%	3.61%	1.7%
			1%	7.24%	2.99%
10 B	34.1%	±3.34%	0.5%	3.61%	2.64%
			1%	7.24%	4.66%
100 B	23.45%	±5.53%	0.5%	3.61%	3.18%
			1%	7.24%	5.76%

[14] 20,000 simulations was enough for the results to converge to at least 2 significant figures.

APPENDIX C: MONTE CARLO SIMULATIONS — IMPLICATIONS OF THE PRICE BAND

Recall that the Sögur Contract sells SGR tokens at a price — the ask price — and buys them back at a lower price — the bid price.

Market price of SGR is free to fluctuate in between the two prices, and only when market price leaves this range does the Contract step in to change the supply of SGR tokens and thereby slow down price movement.

The width of the price band — the percentage distance between the Contract's two prices — gets larger as the SGR economy grows.

The Reserve fund profits the difference between the ask and the bid price every time a token is bought from the Contract and subsequently sold back.

In this Appendix we aim to gauge the amount the Reserve can profit from volatility in SGR price.

Recall the Reserve is used solely to underwrite SGR tokens, and profit in the Reserve is reflected in an equivalent increase in SGR price.

We produced Monte Carlo simulations of fluctuations in SGR market price. The daily returns (the percentage change in SGR price), were drawn from a normal distribution with zero mean and a given standard deviation.

Each Monte Carlo simulation consists of 365 such days. On every simulated day, we check if the new market price is within the Contract's price band. If not, we assume people buy or sell SGR until the Contract's ask or bid price respectively matches the new SGR market price. The supply of SGR is thus adjusted in accordance with the price change.

For each simulation we calculate the amount of money that the Reserve gains from the Contract's buying and selling SGR tokens. Table C.1 below shows the average income — both in absolute and in relative terms — obtained by averaging 20,000 such simulations.

Figure C.1: Example of a Monte Carlo simulation. Simulation starts with SGR market cap at 10B SDR; daily price returns are drawn from a normal distribution with standard deviation 1%

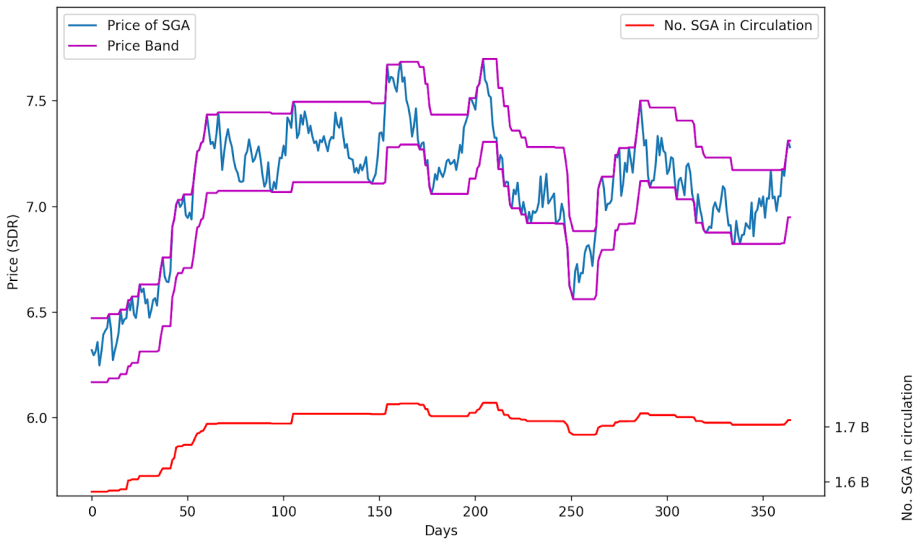


Table C.1: Average gross & net income from 20,000 Monte Carlo simulations of price fluctuations.

Net income is calculated under the assumption that the Reserve undergoes operational costs of 0.15% every time SGR tokens are traded with the Contract. The net income as a percentage of the starting Reserve value gives an indication of how much SGR price would be inflated due to the price adjustment mechanism of Chapter 9.

Simulation parameters:				Simulation results:			
Starting Market Cap	Starting Reserve Value	Starting Width of Price Band	Std of Daily Returns	Price Volatility	Gross Income from Price Band	Net Income from Price Band	Net Income as percentage of starting Reserve
(SDR)	(SDR)	(%)	(%)	(%)	(SDR)	(SDR)	(%)
10M	8.35M	±0.47%	0.5%	3.6%	70K	50K	0.6%
			1%	7.0%	320K	220K	2.6%
			5%	25%	20M	17.7M	200%
100 M	76.3 M	±0.66%	0.5%	3.6%	1M	780K	1%
			1%	7.3%	3.2M	2.5M	3.3%
			5%	30%	56.2M	49.4M	65%
1 B	559M	±1.44%	0.5%	3.6%	3.5M	3.2%	0.6%
			1%	7.3%	11.6M	10.4M	1.8%
			5%	34%	174M	159 M	28%
10 B	3.4 B	±3.34%	0.5%	3.6%	16M	15.3M	0.45%
			1%	7.3%	58.7M	56.1M	1.6%
			5%	38%	1.06B	1.02B	30%
100 B	23.4 B	±5.53%	0.5%	3.6%	95M	92.4M	0.4%
			1%	7.3%	417M	406M	1,7%
			5%	38%	7.1B	6.9B	29%

APPENDIX D: SÖGUR MODEL POINTS

Details of all Model Points of Sögur's monetary model. The majority of points are Genesis Minting Points, where SGR tokens are generated on behalf of SGN holders.

Note:

1. Points 16, 26 & 36 are Vesting Points. Recall these are points where some of Sogur Monetary Technologies' SGN tokens are minted together with SGR tokens needed to catch them up with the rest of the SGN tokens. The SGN-SGR conversion ratio does not change at these points.

2. Points 1, 2,4, 18, 60, 84 & 94 are Target Points. These are points where the parameters of Sögur's reserve ratio function change, without any SGR minted for SGN holders. The SGN-SGR conversion ratio does not change at these points. See Appendix A.

3. There are 107 million SGN tokens in total.

	Market cap	Number of SGR (before)	Minting Amount	Inflation	SGN-SGR Conversion ratio (after)	Reserve ratio (before)	Reserve ratio (after)
#	(SDR)	-	-	%	-	%	%
0	0	0	0	0.00%	0.00	100.00%	100.00%
1	2M	2M	0	0.00%	0.00	100.00%	100.00%
2	12.5M	8.8M	0	0.00%	0.00	80.00%	80.00%
3	37.5M	21.2M	1.01M	4.78%	0.01	80.00%	76.35%
4	131M	55.6M	0	0.00%	0.01	76.35%	76.35%
5	140M	58.5M	1.22M	2.09%	0.03	76.33%	74.77%
6	177M	70.3M	2.28M	3.25%	0.06	74.77%	72.42%
7	256M	92.5M	2.85M	3.09%	0.10	72.42%	70.25%
8	302M	105M	4.29M	4.09%	0.16	70.25%	67.49%
9	396M	127M	4.88M	3.84%	0.22	67.49%	65.00%
10	496M	150M	5.74M	3.84%	0.30	65.00%	62.59%
11	606M	172M	6.31M	3.67%	0.38	62.59%	60.37%
12	724M	194M	7.0M	3.61%	0.48	60.37%	58.27%
13	851M	216M	8.25M	3.81%	0.59	58.27%	56.13%
14	946M	233M	0.67M	0.29%	0.59	56.13%	55.97%
15	1.06B	250M	2.38M	0.95%	0.63	55.97%	55.44%
16	1.08B	253M	6.7M	2.65%	0.63	55.44%	54.01%
17	1.21B	272M	2.79M	1.03%	0.66	54.01%	53.46%

	Market cap	Number of SGR (before)	Minting Amount	Inflation	SGN-SGR Conversion ratio (after)	Reserve ratio (before)	Reserve ratio (after)
#	(SDR)	-	-	%	-	%	%
18	1.40B	296M	0	0.00%	0.66	53.46%	53.46%
19	1.66B	320M	2.3M	0.72%	0.69	52.52%	52.15%
20	1.79B	333M	2.6M	0.80%	0.72	51.75%	51.34%
21	1.92B	345M	2.9M	0.85%	0.75	50.96%	50.53%
22	2.06B	358M	0.17M	0.05%	0.75	50.17%	50.14%
23	2.19B	368M	2.66M	0.72%	0.78	49.79%	49.43%
24	2.33B	380M	3.0M	0.79%	0.82	49.10%	48.71%
25	2.48B	392M	3.14M	0.80%	0.86	48.39%	48.01%
26	2.62B	402M	9.1M	2.28%	0.86	47.75%	46.69%
27	2.84B	421M	3.4M	0.81%	0.89	46.36%	45.99%
28	3.00B	433M	0.3M	0.08%	0.89	45.72%	45.69%
29	3.15B	442M	3.09M	0.70%	0.93	45.43%	45.11%
30	3.32B	453M	3.51M	0.77%	0.96	44.86%	44.52%
31	3.49B	464M	3.64M	0.78%	1.00	44.28%	43.94%
32	3.74B	478M	3.74M	0.78%	1.04	43.61%	43.28%
33	3.99B	493M	3.84M	0.78%	1.08	42.97%	42.64%
34	4.25B	507M	3.9M	0.78%	1.12	42.35%	42.03%
35	4.51B	521M	4.0M	0.77%	1.16	41.76%	41.44%
36	4.87B	538M	12.4M	2.31%	1.16	41.09%	40.16%
37	5.13B	556M	4.5M	0.81%	1.20	40.02%	39.69%
38	5.42B	570M	4.32M	0.76%	1.24	39.48%	39.18%
39	5.71B	583M	4.38M	0.75%	1.29	38.98%	38.69%
40	6.01B	596M	4.4M	0.75%	1.33	38.50%	38.21%
41	6.32B	610M	9.07M	1.49%	1.41	38.03%	37.48%
42	7.22B	644M	8.7M	1.36%	1.49	36.99%	36.49%
43	8.20B	677M	13.7M	2.03%	1.62	36.06%	35.34%
44	9.3B	714M	15.1M	2.12%	1.76	34.97%	34.24%
45	10.5B	752M	15.9M	2.11%	1.91	33.93%	33.23%
46	11.8B	791M	7.62M	0.96%	1.98	32.98%	32.67%
47	13.1B	821M	6.26M	0.76%	2.04	32.45%	32.21%
48	14.4B	849M	6.18M	0.73%	2.10	32.01%	31.78%

	Market cap	Number of SGR (before)	Minting Amount	Inflation	SGN-SGR Conversion ratio (after)	Reserve ratio (before)	Reserve ratio (after)
#	(SDR)	-	-	%	-	%	%
49	15.8B	877M	6.33M	0.72%	2.16	31.61%	31.39%
50	17.2B	905M	6.55M	0.72%	2.22	31.25%	31.02%
51	18.8B	933M	6.75M	0.72%	2.28	30.91%	30.68%
52	20.4B	961M	6.97M	0.72%	2.35	30.60%	30.38%
53	22.1B	989M	7.18M	0.73%	2.42	30.32%	30.11%
54	23.9B	1.02B	7.4M	0.73%	2.48	30.09%	29.87%
55	25.8B	1.05B	7.6M	0.73%	2.56	29.87%	29.66%
56	27.7B	1.08B	7.8M	0.73%	2.63	29.66%	29.44%
57	29.9B	1.1B	8.0M	0.73%	2.70	29.44%	29.23%
58	32.1B	1.13B	8.2M	0.73%	2.78	29.23%	29.02%
59	34.5B	1.16B	8.4M	0.73%	2.86	29.02%	28.81%
60	34.7B	1.17B	0	0.00%	2.86	28.81%	28.81%
61	37.3B	1.19B	8.6M	0.72%	2.94	28.56%	28.35%
62	40.3B	1.22B	8.8M	0.72%	3.02	28.11%	27.91%
63	43.5B	1.25B	9.1M	0.72%	3.11	27.67%	27.47%
64	47.0B	1.28B	9.3M	0.72%	3.19	27.24%	27.05%
65	50.6B	1.31B	9.5M	0.72%	3.28	26.83%	26.63%
66	54.6B	1.34B	9.7M	0.72%	3.37	26.42%	26.23%
67	58.7B	1.37B	9.9M	0.72%	3.47	26.02%	25.84%
68	63.2B	1.41B	10M	0.72%	3.56	25.64%	25.45%
69	67.9B	1.44B	10.3M	0.72%	3.66	25.26%	25.08%
70	73.0B	1.47B	10.5M	0.72%	3.76	24.90%	24.72%
71	78.3B	1.5B	10.7M	0.72%	3.86	24.54%	24.37%
72	84.0B	1.53B	10.9M	0.72%	3.96	24.20%	24.03%
73	90.1B	1.56B	11.1M	0.71%	4.06	23.86%	23.69%
74	104B	1.62B	20.3M	1.26%	4.25	23.35%	23.06%
75	121B	1.69B	22.9M	1.36%	4.47	22.74%	22.44%
76	140B	1.76B	24.1M	1.37%	4.69	22.15%	21.85%
77	162B	1.83B	15.44M	0.85%	4.84	21.61%	21.43%
78	186B	1.89B	13.8M	0.73%	4.97	21.21%	21.06%
79	213B	1.95B	13.8M	0.71%	5.10	20.87%	20.72%

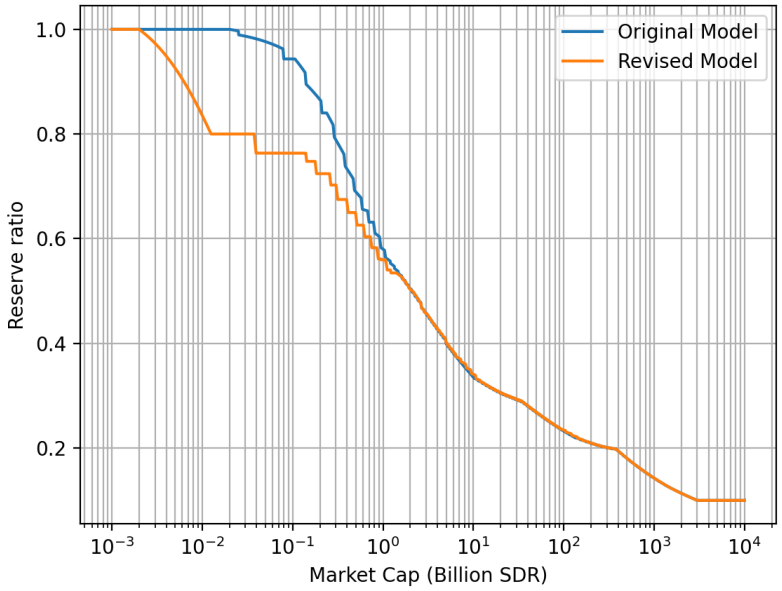
	Market cap	Number of SGR (before)	Minting Amount	Inflation	SGN-SGR Conversion ratio (after)	Reserve ratio (before)	Reserve ratio (after)
#	(SDR)	-	-	%	-	%	%
80	243B	2.01B	14.2M	0.70%	5.23	20.56%	20.42%
81	277B	2.08B	14.6M	0.70%	5.37	20.30%	20.16%
82	315B	2.14B	15.1M	0.70%	5.51	20.09%	19.95%
83	357B	2.21B	15.71M	0.71%	5.65	19.95%	19.81%
84	378B	2.25B	0	0.00%	5.65	19.81%	19.81%
85	415B	2.28B	16M	0.70%	5.80	19.26%	19.12%
86	494B	2.34B	16.6M	0.71%	5.96	18.16%	18.03%
87	589B	2.41B	17.1M	0.71%	6.12	17.12%	17.00%
88	702B	2.47B	17.58M	0.71%	6.28	16.13%	16.01%
89	835B	2.53B	18.0M	0.71%	6.45	15.20%	15.09%
90	995B	2.60B	18.5M	0.71%	6.62	14.32%	14.22%
91	1.18T	2.66B	18.9M	0.71%	6.80	13.49%	13.40%
92	1.41T	2.72B	19.3M	0.71%	6.98	12.72%	12.63%
93	1.67T	2.77B	8.15M	0.29%	7.06	12.01%	11.97%
94	3.0T	2.91B	0	0.00%	7.06	10.00%	10.00%

APPENDIX E: SUMMARY OF CHANGES IN THE SEPTEMBER 2020 REVISED MODEL

In this paper we have described the Monetary Model following its September 2020 update. Here we describe the main changes with respect to the original model.

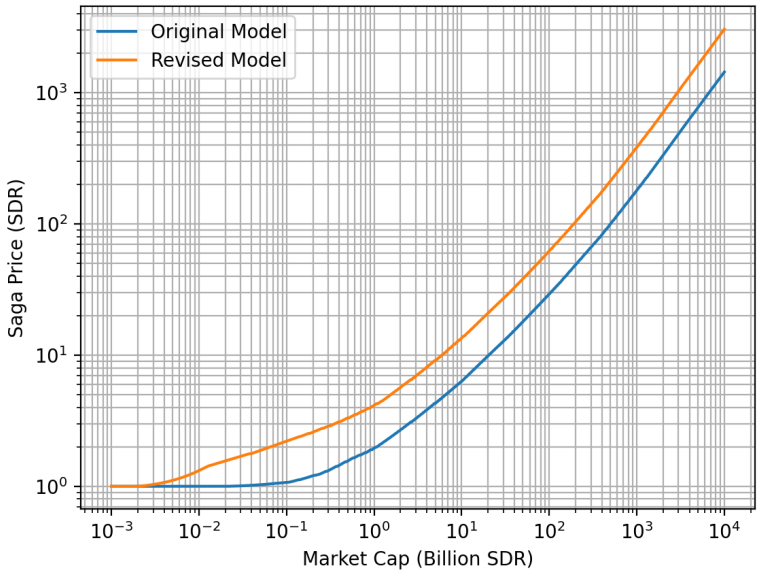
In the original model the reserve ratio departed from 100% at a market cap of 20M SDR. This market cap threshold was lowered to 2M SDR, with the reserve ratio reaching 80% at a market cap of 12.5M SDR. The reserve ratio curve of the new model meets the original model's curve at a market cap of ~1B SDR. From this point onwards the two curves behave the same, ensuring that in the long term SGR maintains the same properties it had under the old model while it offers higher volatility when the economy is still small. This higher volatility may serve as an incentive for early adopters in the short term.

Figure E.1: Comparison between the reserve ratio of the original and the revised models



As a result of the reserve ratio changes, the SGR price in the revised model starts increasing at a market cap of 2M SDR (instead at a market cap of 20M SDR in the old Model), reaching a price of 4.15SDR at a market cap of 1B SDR (instead of 1.95SDR in the old model).

Figure E.2: Comparison between SGR price in the original and the revised models



The SGN mechanism has been updated such that the SGN to SGR conversion ratio is lower compared to the original model, while the SGN conversion value in SDR terms remains unchanged (due to the fact that the SGR price in SDR terms under the new model is higher). Altogether, the maximal number of SGR tokens minted for each SGN token is reduced from 15 SGR per SGN token to 7.057.

The revised model also changed the price band behaviour. In both models the price band starts at a width of ±0.15%. While in the original model the price band width remained fixed until a market cap of 1B SDR, in the revised model the price band width starts to increase from the beginning. This change is came as we now know there is already an efficient secondary market for SGR.